Prove by induction: \[1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = 2 - \frac{1}{2^n}\]

We begin with the base case. Since \[1 + \frac{1}{2} = 2 - \frac{1}{2}\], the statement holds for \(n=1\).

We now consider the inductive step. Suppose \[1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = 2 - \frac{1}{2^n}\]. We seek to show \[1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} + \frac{1}{2^{n+1}} = 2 - \frac{1}{2^{n+1}}\].

Note that, by our inductive hypothesis and some algebra,

\[
1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} + \frac{1}{2^{n+1}} = (2 - \frac{1}{2^n}) + \frac{1}{2^{n+1}} = 2 - \left(\frac{1}{2^n} - \frac{1}{2^{n+1}}\right) = 2 - \frac{1}{2^n} \left(1 - \frac{1}{2}\right) = 2 - \frac{1}{2^n} \cdot \frac{1}{2} = 2 - \frac{1}{2^{n+1}}
\]

This completes the inductive step.

Therefore \(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = 2 - \frac{1}{2^n}\) must hold for all \(n \in \mathbb{N}\).
"Regular" Induction

\[ 1 \ 2 \ 3 \ 4 \ 5 \ \cdots \ n \ n+1 \]

Variant on HW1, Q2

\[ m \ m+1 \ m+2 \ \cdots \ n \ n+1 \]

Strong Induction

Base Case: \( P_1 \) is true

Inductive Step: If \( P_1, P_2, P_3, \ldots, P_n \) are all true, then \( P_{n+1} \) is true

Fix \( n \in \mathbb{N} \).
Assume \( n \in \mathbb{N} \).

Let \( n \in \mathbb{N} \).

\[ \{ \text{mean the same thing} \} \]
HW 1
Q5 (i)
Prove \( |a| \geq 0 \).

Case 1: Assume \( a \geq 0 \).

By definition of the absolute value,
\( |a| = a \), which is nonnegative by assumption.

Case 2: Assume \( a < 0 \).

By definition of the absolute value, \( |a| = -a \).

Recall from Exercise 2(1), since \( a \leq 0 \), as shown in class, \(-0 = -a\). Since \(-0 = 0\), we have \( 0 \leq -a = |a| \).

If the problem asked instead for you to prove this using the defn of an ordered field, you would need to show \(-0 = 0\).

In the defn of an ordered field, for any \( a \in F \), \(-a\) is the number so that \( a + (-a) = 0\).
However, by defn of 0 as additive identity, 
\( \text{0} + \text{0} = \text{0} \).

Thus 0 is its own additive inverse, which means \( \text{0} = -\text{0} \).

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**Intuitive understanding of supremum:**

\( S = \{0, 1\} \)

---

\( \text{0} \quad 1 \quad \frac{3}{2} \quad 2 \)

**Does this set have a maximum?** No.
**Does this set have a supremum?** Yes, 1

*Least Upper Bound* (LUB)

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\( S = \{0, 1\} \)

---

**Does this set have a maximum?** Yes, 1
**Does this set have a supremum?** Yes, 1

---

\( -1 \quad -\frac{1}{2} \quad 0 \quad 1 \)

**Does this set have a minimum?** No
**Does this set have an infimum?** Yes, 0

*Greatest Lower Bound* (GLB)
\[ S = [0, 1] \]

Does this set have a minimum? Yes, 0

Does this set have an infimum? Yes, 0

Extended Real Numbers

\[ \overline{\mathbb{R}} = \mathbb{R} \cup \{ \pm \infty \} \]

\[ \mathbb{C} = \{ x + iy : x, y \in \mathbb{R}^2, i = \sqrt{-1} \} \]

\[ \mathbb{Q} = \{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \} \]

Fact: \( \pm \infty \) and \( \mp \infty \) are not in \( \overline{\mathbb{R}} \).