Practice Quiz 1

4. (a) $|b| < a \iff -a < b < a$

(b) $|a-b| < c \iff b-c < a < b+c$

"$a$ and $b$ are less than distance $c$ apart"

(c) $|a-b| \leq c \iff b-c \leq a \leq b+c$

② Prove $0<1$.

Proof:

Claim: $\exists a \in F$ s.t. $a>0$

Proof of Claim: $\exists 1 \in F$ s.t. $1 \neq 0$. By totality, either $1>0$ or $1<0$ and Thm 1(i) ensures $0=-0<1$.

By (v), $\frac{1}{a}>0$.

By (iv) $a \cdot 0 < \frac{1}{a} \cdot a \implies 0<1$.

Additional Proof:

Step 1: Prove $0 \leq 1$.

Step 2: Prove $0 \neq 1$. This follows by definition of
foll

\[ \begin{align*}
& \text{the multiplicative identity } 1. \\
& \text{Let } S \text{ be a nonempty subset of } \mathbb{R} \text{ that is bold above.}
\end{align*} \\
\[
\begin{align*}
\text{WTS: } \sup(S) \in S \Rightarrow \sup(S) = \max(S). \\
\text{It suffices to show that } \sup(S) \text{ is the largest element in } S. \\
\text{• } \sup(S) \in S \text{ by assumption} \\
\text{• Let } t \in S.
\end{align*}
\]

\[ \begin{align*}
\text{WTS } \text{ if } \sup(S) \notin S, \text{ then } S \text{ doesn't have a max.} \\
\text{Suppose } \sup(S) \notin S \\
\text{Let } t \in S. \text{ It suffices to show } \exists \ t \in S \text{ s.t. } t > t_0. \text{ This will ensure there is no largest element in the set.}
\end{align*} \]
Assume, for the sake of contradiction that \( \not\exists t \in S \) s.t. \( t > t_0 \). In other words, for all \( t \in S \), \( t \leq t_0 \).

Thus \( \text{sup}(S) \in S \), which is a contradiction.

Q8
Prop: Every nonempty subset \( S \) of \( \mathbb{R} \) that is bounded below has an infimum.

(a) WTS \( -S = \{ -s : s \in S \} \) is bdd above.

Pf:
By definition, \( -S \) is bdd above if \( \exists M \in \mathbb{R} \) s.t. \( \forall t \in -S \), \( t \leq M \).

Since \( S \) is bounded below, by defn, \( \exists m \in \mathbb{R} \) s.t. \( \forall s \in S \), \( m \leq s \).

(b) WTS \( -S \) has a supremum
\[ \text{Step 1: } WTS - \sup(-S) \text{ is a lower bound of } S. \]

Since \( \sup(-S) \) is an upper bound for \(-S\),
\[-s \leq \sup(-S) \quad \forall s \in S \iff -\sup(-S) \leq s \quad \forall s \in S. \]

Thus, \(-\sup(-S)\) is a lower bound of \( S \).

\[ \text{Step 2: } WTS - \sup(-S) \text{ is greatest lower bound of } S. \]