Want to prove: \( n! > n^2 \) \( \forall n \geq 4 \)

Proof by induction

Base \( n = 4 \) \( 4! = 24 > (4)^2 = 16 \)

Inductive Step (a little different assumption for strong ind)

Fix some \( n_0 \geq 4 \) & assume \( n! > n^2 \)

\( \forall \, \forall \, n \leq n_0 \)

Regular ind.

\( \checkmark \)

Strong ind.

\( \checkmark \)

\( \checkmark \)

\( \checkmark \)

\( \checkmark \)

\( ? \)
Want to show: \((n+1)! \geq (n+1)^2\)

\[(n+1)! = (n+1) \cdot n! \geq (n+1) \cdot n^2 \geq (n+1) \cdot (n+1)\]

Inductive assumption: \(n! > n^2\)

By part (a), we know when \(n \geq 2\), \(n^2 > n+1\)

4.2 (totality) \(\forall a, b \in \mathbb{R}\) either have:
\[a \leq b\]
OR
\[b \geq a\]
totality with $0 \in \mathbb{R}$, $\forall a \in \mathbb{R}$, either $a \leq 0$ or $a \geq 0$.

2 cases:

Case (i) $a \geq 0$

wTn $\frac{a^2}{n} = a \cdot a \geq 0$ ?

axioms

$\begin{align*}
\text{ordered } & a \geq b \\
\text{field (axiom in class)} & ca \geq cb \\
& ac \geq bc
\end{align*}$

$a \geq 0 \implies a \cdot a \geq 0$

Case (ii) $a \leq 0$
\[ a \leq 0 \Rightarrow -a \geq 0 \quad \text{(by Part (a))} \]

Use previous case

\[ (-a)(-a) \geq 0 \]

\[ \Rightarrow (-1)(-1)a^2 \]

\[ \Rightarrow a^2 \]

5.2 \hspace{1cm} \text{WTS} \hspace{0.5cm} \forall a, b \in \mathbb{R} \hspace{0.5cm} |ab| = |a| \cdot |b|

\text{Defn} \hspace{1cm} | \cdot | : \mathbb{R} \to \mathbb{R}_{\geq 0}

\[ |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} \]
Let's fix some \( a, b \in \mathbb{R} \)

\[
|ab| = |a| \cdot |b|
\]  

(by totality of ordered field applied to 0)

3 cases

(i) both nonnegative

by ordered field axioms \( a \geq 0 \) & \( b \geq 0 \)  
\[ \Rightarrow \quad ab \geq 0 \]

which means  
\[ |ab| = ab = a \cdot b \]

(ii) both non-positive

Part 4(a) \( \Rightarrow \quad -a \geq 0, -b \geq 0 \)

by case (i) above,  
\[ |ka|(-b) = |a| |-b| = |a| \cdot |b| \]

(since assumed neg.)
You all want you need I want you:

\[ |a| = |a| \quad \forall a \in \mathbb{R} \]

\[ |ab| = \left| (-1)^2 ab \right| = |(-a)(-b)| = |-a| |-b| = |a| |b| \]

\[ (-1)^2 = 1 \quad \text{Identity} \]

\[ \text{Prev slide} \]

(iii) Case WLOG \( a \geq 0 \)
\[ b \leq 0 \]
\[ |a| = a \]
\[ |b| = -b \]

\[ a \geq 0, b \leq 0 \implies a \geq 0, -b \geq 0 \quad \text{Field axioms} \]
\[ \implies -ab \geq 0 \]
\[ \implies ab \leq 0 \]

\[ \text{4. part a} \]

\[ \text{go back to def of } |x| \]
Direct Proof

\[ P \Rightarrow Q \]

Contradiction

\[ (\neg P \land Q) \]

Contra-positive

\[ \neg Q \Rightarrow \neg P \]