This is a closed-book and closed-note examination. Calculators are not allowed. Please show your work in the space provided. Scratch paper is not permitted. If you continue a problem on the back of a page, please write “continued on back”. Partial credit will be given for partial answers. You have 1 hour and 15 minutes.

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Consider a sequence $s_n$.

(a) State the definition of $\lim \sup s_n$. (6 points)

(b) If $\lim \sup |s_n| < +\infty$, prove that $s_n$ is a bounded sequence. (7 points)

(c) If $s_n$ is a bounded sequence, prove that $\lim \sup |s_n| < +\infty$. (7 points)
Question 4 (20 points)

(a) State the definition of what it means for a series to converge. (6 points)

(b) Consider a sequence of real numbers \( b_n \). Prove that \( \sum_{n=1}^{\infty} |b_n| \) either diverges to \(+\infty\) or converges. (7 points)

(c) Use the Cauchy Criterion to prove that, if \( \sum_{n=1}^{\infty} |b_n| \) is convergent, then \( \sum_{n=1}^{\infty} b_n \) is also convergent. (7 points)

(Do not use the Comparison Test theorem for convergent series from the textbook, since we did not discuss that theorem in class.)
Question 5 (20 points)

Lightning Round!
You do not need to show your work or justify your answers.

(a) Consider the sequence \( s_n = (-1)^n + \frac{1}{n} \).

(i) Give an example of a monotone subsequence. (3 points)

(ii) What is the set of subsequential limits of \( s_n \)? (3 points)

(iii) What is \( \limsup_{n \to \infty} s_n \)? (2 points)

(iv) What is \( \liminf_{n \to \infty} s_n \)? (2 points)

(b) Determine if the following statements are true or false. If they are false, provide a counterexample.

(i) Suppose \( s_n \) has a subsequence \( s_{n_k} \) so that \( \lim_{k \to +\infty} s_{n_k} = +\infty \). Then \( \limsup_{n \to +\infty} s_n = +\infty \). (5 points)

(ii) Suppose \( s_n \) has a subsequence \( s_{n_k} \) so that \( \lim_{k \to +\infty} s_{n_k} = 0 \). Then \( \limsup_{n \to +\infty} s_n = 0 \). (5 points)