Math 260L: Optimal Transport and Gradient Flows

Prof. Katy Craig

Notetaker? Email me:

Course goals
① pleasant distraction
② transition from coursework to research
   = DOT Wiki
   = Do Want more?
③ math goals
   A. What is the optimal transport problem? How does duality help us solve this problem?
   B. What is a Wasserstein gradient flow? What is the relationship between convexity of an energy and well-posedness of a gradient flow?
   C. Exposure to interplay between convex analysis, PDE, probability, functional analysis, geometry, and optimization.
Optimal Transport

Gaspar Monge, 1781
"On cuttings and embankments"

Q: How can we rearrange the dirt in $\mu$ to look like $v$ in the most efficient way?

Q: Why do we care?

A: The amount of effort it takes to rearrange one pile of dirt to look like another provides a notion of distance that turns out to be useful in PDE, machine learning, geometry.
If we measure distance in "usual way" (\(L^p\) norms)... 

\[
\int |\mu_1(x) - \mu_2(x)| \, dx = 4 \\
\int |\mu_1(x) - \mu_3(x)| \, dx = 4
\]

Moral: Whenever the independent variable \(x\) has an interpretation in terms of location in physical space, OT can provide a more natural notion of distance.

Impact of OT over past 20 years:
Machine Learning

\[(1-\alpha)\mu + \alpha\nu, \alpha \in [0, 1]\]

"usual" interpolation,

linear interpolation

\[\text{PDE: Two Fields medals}\]

Villani (2010), Figalli (2018)

\[\text{Geometry: novel characterization of}\]

Ricci curvature in terms of

convexity of entropy functional

\[\text{Monge, } \mu\]

\[\text{Kantorovich, } \nu\]

\[\text{OT interpolation}\]

"displacement interpolation"

rearranging pixels like

grains of dirt

\[\text{[Peyré, Papadakis, Oudet 2013]}\]