Math 134: Homework 6.5

Extra practice for midterm, not to be turned in

Questions followed by * are to be turned in. Questions without * are extra practice. At least one extra practice question will appear on each exam.

You should solve these problems without the aid of a computer/calculator, as you will not have one on the exams. Feel free to use one to check your answers, though.

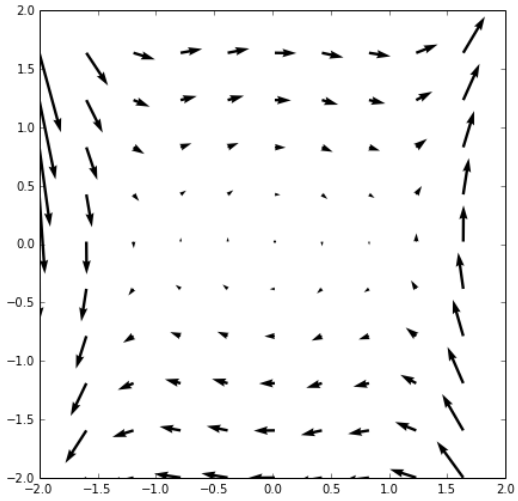
Question 1 (Strogatz 6.5.1)

Consider the system $\ddot{x} = x^3 - x$.

(a) Find a conserved quantity.

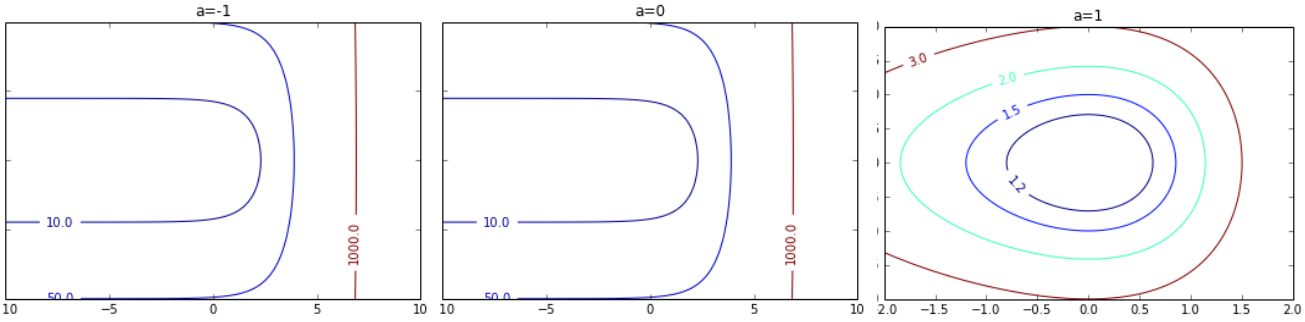
(b) Find all the fixed points and classify them.

(c) Sketch the phase portrait, including nullclines, fixed points, and sample solution trajectories. You may use the below vector field as a guide.



Question 2 (Strogatz 6.5.3)

Find a conserved quantity for the system $\ddot{x} = a - e^x$ and sketch the phase portrait for $a = -1$, $a = 0$, and $a = 1$. Include nullclines, fixed points, and sample solution trajectories. Below, are contour plots of the function $\frac{1}{2}(y)^2 - ax + e^x$ for $a = 1$, $a = 0$, and $a = -1$, which should help you draw the phase portraits.



Question 3 (Similar to Strogatz 6.5.15)

In this question, we return to the problem of a bead on a rotating hoop. Recall that the bead's motion is governed by

$$mr\ddot{\phi} = -b\dot{\phi} - mg\sin\phi + mr\omega^2\sin\phi\cos\phi.$$

Previously, we could only treat the overdamped limit. Now, we will consider the undamped case $b = 0$.

(a) Show that the equation can be nondimensionalized to $\phi'' = \sin\phi(\cos\phi - \gamma^{-1})$ where $\gamma = r\omega^2 g$ as before, where prime denotes differentiation with respect to dimensionless time $\tau = \omega t$.

(b) Find all fixed points and classify them for all qualitatively different values of γ.

(c) What does the classification of the fixed points imply about the physical motion of the bead?

Question 4 (Strogatz 6.6.7)

(a) Show that the system $\ddot{x} + x\dot{x} + x = 0$ is reversible. (Hint: you need to interchange the usual roles of x and y.)

(b) Plot the phase portrait, including nullclines, fixed points, and sample solution trajectories. You may use the below vector field as a guide.