Math 134: Homework 7.5

extra practice for final: not to be turned in

Questions followed by * are to be turned in. Questions without * are extra practice. At least one extra practice question will appear on each exam.

You should solve these problems without the aid of a computer/calculator, as you will not have one on the exams. Feel free to use one to check your answers, though.

Question 1 (Strogatz 7.2.10)

Show that the system $\dot{x} = y - x^3$, $\dot{y} = -x - y^3$ has no closed orbits by constructing a Liapunov function $V = ax^2 + by^2$ with suitable a, b.

Question 2 (Strogatz 7.3.1)

Consider $\dot{x} = x - y - x(x^2 + 5y^2)$, $\dot{y} = x + y - y(x^2 + y^2)$.

(a) Classify the fixed point at the origin.

(b) Rewrite the system in polar coordinates, using $r\dot{r} = x\dot{x} + y\dot{y}$ and $\dot{\theta} = (x\dot{y} - y\dot{x})/r^2$.

(c) Determine the circle of maximum radius r_1, centered at the origin, on which all trajectories have a radially *outward* component.

(d) Determine the circle of minimum radius r_2, centered at the origin, on which all trajectories have a radially *inward* component.

(e) Prove that the system has a closed orbit somewhere in the trapping region $r_1 \leq r \leq r_2$.

Question 3 (Strogatz 8.1.6)

Consider the system $\dot{x} = y - 2x$, $\dot{y} = \mu + x^2 - y$.

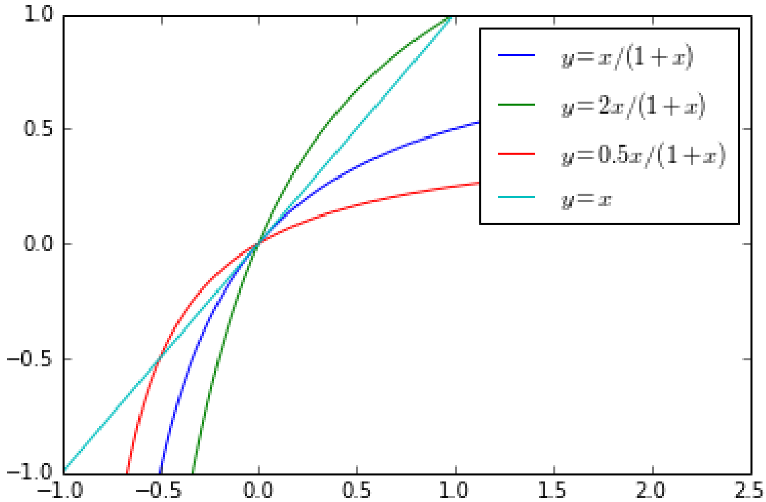
(a) Sketch the nullcines. (Don't worry about sketching the vector field along the nullclines for this part.)

(b) Find and classify the bifurcations that occur as μ varies.

(c) Sketch the phase portraits for all qualitatively different values of μ. You may use pplane to help.

Question 4 (Strogatz 8.1.7)

Find and classify all bifurcations of the system $\dot{x} = y - x$, $\dot{y} = -by + x/(1 + x)$ for $b > 0$ and $x > -1$. The following picture may be helpful.



Question 5 (Strogatz 8.1.8)

In Section 3.5, we derived the following dimensionless equation for the motion of a bead on a rotating hoop:

$$\epsilon \frac{d^2\phi}{d\tau^2} = -\frac{d\phi}{d\tau} - \sin\phi + \gamma \sin\phi \cos\phi.$$

Here $\epsilon > 0$ is proportional to the mass of the bead, $\gamma > 0$ is related to the spin rate of the hoop. Previously we restricted our attention to the overdamped limit $\epsilon \to 0$. Now let $\epsilon = 1$. Find and classify all bifurcations that occur as γ varies.