Math 164: Homework 3

Due Friday April, 17th

Questions followed by * are to be turned in. Questions without * are extra practice. At least one extra practice question will appear on each exam.

Question 1 (Similar to Textbook Problem 3.1.2)

Consider the set defined by the constraints $x_2 - x_1 = 0$, $x_1 \leq 1$, and $x_2 \leq 1$. At each of the following points determine the set of feasible directions: $x_a = (0,0)^T$, $x_b = (1,1)^T$, $x_c = (0.5, 0.5)^T$.

Question 2* (Similar to Textbook Problem 4.1.1)

Consider the problem

$$\begin{aligned} \text{minimize } & f(x) \ , \\ \text{subject to } & x_1 + 2x_2 + 4x_3 = 8, \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \ . \end{aligned}$$

(a) Find the set of all feasible directions at points $x_a = (0,0,2)^T$, $x_b = (2,1,1)^T$, $x_c = (6,1,0)^T$.

(b) Using part (a), verify that $p = (-4, 0, 1)^T$ is a feasible direction for $x_c = (6,1,0)^T$. Then find an upper bound on the step length α so that $x_c + \alpha p$ is a feasible point.

Question 3* (Similar to Textbook Problem 4.1.1)

Consider the linear program

$$\begin{aligned} \text{minimize } & f(x) \ , \\ \text{subject to } & x_1 - x_2 \leq 1, \\ & x_1 + x_2 \leq 1 \ , \\ & x_1 \geq 0 \ . \end{aligned}$$

For the following choices of $f(x)$, solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show that none exists: (a) $f(x) = -x_1$, (b) $f(x) = x_2$, (c) $f(x) = -x_1 - x_2$.
Do any of the functions have more than one global minimizer?

Question 4 (Similar to Textbook Problem 4.1.1)

Fix $a > 0$. Solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show none exists.

$$\begin{aligned} \text{minimize } & f(x) = x_1 - 2x_2 \ , \\ \text{subject to } & x_1 + x_2 \leq a \ , \\ & x_1 \geq 0 \ , \\ & x_2 \geq 0 \ . \end{aligned}$$

Question 5 (Similar to Textbook Problem 4.1.1)

Solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show that none exists.

$$\begin{aligned} \text{minimize } & -x_1 + 2x_2 \ , \\ \text{subject to } & 5x_1 + 2x_2 \geq 10 \ , \\ & 2x_1 + 3x_2 \leq 40 \ , \\ & x_1 \leq 15 \ , \\ & x_2 \leq 15 \ . \end{aligned}$$

Question 6* (Similar to Textbook Problem 4.1.1)

Solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show that none exists.

$$\begin{aligned} \text{minimize } & -x_1 - x_2 \ , \\ \text{subject to } & x_2 - x_1 \geq 0 \ , \\ & x_2 - 2x_1 \geq 2 \ , \\ & x_1 \geq 0 \ , \\ & x_2 \geq 0 \ . \end{aligned}$$

Question 7* (Similar to Textbook Problem 4.1.1)

Solve the linear program graphically, i.e. find a global minimizer for $f(x)$ or show that none exists.

$$\begin{aligned} \text{minimize } & \pi x_1 + e x_2 \ , \\ \text{subject to } & x_1 + x_2 \leq 6 \ , \\ & x_2 - x_1 \geq 3 \ , \\ & 2x_1 - x_2 \geq 2 \ , \\ & x_1 \geq 0 \\ & x_2 \geq 0 \ . \end{aligned}$$

no question 8?

Question 9* (Similar to Textbook Problem 4.2.2)

Convert the following linear program to standard form:

$$\begin{aligned} \text{minimize } & z = x_1 - 5x_2 - 7x_3 \ , \\ \text{subject to } & 3x_1 - x_2 + 9x_3 \geq 7 \ , \\ & 5x_1 + 0x_2 - 3x_3 = 1 \ , \\ & 7x_1 + 5x_2 + 5x_3 \leq 9 \ , \\ & x_1 \geq -2 \ , \\ & x_2, x_3 \text{ free.} \end{aligned}$$

Question 10 (Similar to Textbook Problem 4.2.3)

Convert the linear program in Question 5 to standard form.