Math 164: Homework 4.5

Not to be turned in – extra practice for Midterm 1

Question 1 (Similar to Textbook Problem 4.4.5)

Let $\{d_1, \ldots, d_k\}$ be directions of unboundedness for the constraints $Ax = b$, $x \geq 0$. Prove that

$$d = \sum_{i=1}^{k} \alpha_i d_i \text{ with } \alpha_i \geq 0$$

is also a direction of unboundedness for these constraints.

Question 2 (Similar to Textbook Problem 4.4.6)

Consider the linear program

$$\begin{aligned} \text{minimize } & z = 2x_1 - 3x_2 \ , \\ \text{subject to } & 6x_1 + 8x_2 \leq 24, \\ & x_2 - 2x_1 \leq 2 \ , \\ & x_1, x_2 \geq 0 \ . \end{aligned}$$

Represent the point $x = (1, 1)^T$ as a convex combination of extreme points, plus, if applicable, a direction of unboundedness. Find two different representations.

Question 3 (Similar to Textbook Problem 4.4.8)

Suppose that a linear program with bounded feasible region has l optimal extreme points $v_1, \ldots, v_l$. Prove that a point is optimal for the linear program if and only if it can be expressed as a convex combination of these optimal extreme points.

Question 4 (Textbook Problem 5.2.7)

Prove that the set of optimal solutions to a linear program is a convex set.

Question 5

Consider the linear program:

$$\begin{aligned} \text{minimize } & x_1 - x_2 \ , \\ \text{subject to } & x_1 + x_2 \leq 5 \ , \\ & x_1 + 2x_2 \leq 6 \ , \\ & x_1, x_2 \geq 0 \ . \end{aligned}$$

(a) Put the linear program into standard form by introducing slack variables $\{x_3, x_4\}$.

(b) Show that $[0, 3, 2, 0]^T$ is a basic feasible solution. What is the corresponding set of basic variables?

(c) Is $[0, 3, 2, 0]^T$ the minimizer? If no, give a feasible direction which lowers the value of the objective function. If yes, show that there is no feasible direction which lowers the value of the objective function. **Hint: use the characterization of feasible directions at a b.f.s. that we derived in class.**

(d) Solve the linear program graphically and compare you answer with part (c).