1 Derivative Formulas

The following two properties are known as the linearity of the derivative:
\[
\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)
\]

“The derivative of the sum is the sum of the derivatives.” or “You can take the derivative term by term.”

\[
\frac{d}{dx}[cf(x)] = cf'(x)
\]

“You can pull out constants.”

Other Rules:

• Power Rule: \( \frac{d}{dx}(x^n) = nx^{n-1} \)
  
  What does this say about the derivative of a constant?

• Product Rule: \( \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \)

• Quotient Rule: \( \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \)
  
  “LodiHi-HidiLo all over LoLo.”

• Chain Rule: \( \frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \)

• \( \frac{d}{dx}(e^x) = e^x \)

• \( \frac{d}{dx}\ln(|x|) = \frac{1}{x} \)

• \( \frac{d}{dx}\sin(x) = \cos(x) \quad \frac{d}{dx}\cos(x) = -\sin(x) \quad \frac{d}{dx}\tan(x) = \sec^2(x) \)

• \( \frac{d}{dx}\csc(x) = -\csc(x)\cot(x) \quad \frac{d}{dx}\sec(x) = \sec(x)\tan(x) \quad \frac{d}{dx}\cot(x) = -\csc^2(x) \)

• \( \frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \)

You will need to memorize these! You spent the entire last course mastering how to calculate a derivative, and this course will be all about undoing your hard work. :( So you still have to have all of these memorized.
2 The Antiderivative

The antiderivative of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$.

Examples:
If $f'(x) = 2x^2$, then $f(x) =$ If $f'(x) = x^3$, then $f(x) =$
If $f(x) = \sin(x)$, then $F(x) =$

Question: Are antiderivatives unique?

- Power Rule: If $f'(x) = x^n$, then what is $f(x)$?
  
  In other words, $(n \neq -1)$
  
  \[
  \int x^n dx = 
  \]

- If $f(x) = \frac{1}{x}$, what is $F(x)$?
  
  In other words,
  
  \[
  \int \frac{dx}{x} = 
  \]

- If $f(x) = e^x$, what is $F(x)$? In other words,
  
  \[
  \int e^x dx = 
  \]

- If $f''(x) = e^x + 6x$, what is $f(x)$?

Fill out the following antiderivative formulas if possible:

\[
\int \sin(x) dx = \quad \int \cos(x) dx = \quad \int \sec^2(x) dx = \\
\int \tan(x) dx = \quad \int \ln(x) dx =
\]

etc... Everything on the first page gives you an integration formula. But some seemingly common functions don’t have an obvious derivative while some seemingly obscure functions do!