1 Exponent Formulas

“e.g. powers of $x^n$ (x is a variable) or powers of a number

\[ x^n = \underbrace{x \cdot x \cdots x}_{n \text{ copies}}, \text{x is the base, } n \text{ is the exponent} \]

**Exponent Laws** (Try to prove the last two!):

- $x^0 = 1$
- $x^{-n} = \frac{1}{x^n}$
- $x^{\frac{1}{n}} = \sqrt[n]{x}$
- $x^m \cdot x^n = x^{m+n}$
- $(x^m)^n = x^{mn}$

These can be used to derive the following rules (Try deriving them!):

- $\frac{x^m}{x^n} = x^{m-n}$
- $x^{m/n} = (\sqrt[n]{x})^m$
- $x^{m/n} = \sqrt[n]{x^m}$
- $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

2 Quadratic Formula: Solving $f(x) = 0$

When $f(x) = ax^2 + bx + c = 0$, you must solve for $x$ either by factoring or by using the quadratic formula.

**Quadratic formula:** If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

*A comment on the quadratic equation from Stephen Colbert:* "What you are feeling [as you look at the quadratic equation above] is your body rejecting an idea that is trying to make you learn it. Don’t fight the confusion. That’s just your body scabbing over in a desperate attempt to protect you from that unnatural co-mingling of numbers and letters up there. You can’t add it, and you can’t read it. Useless."  

**Disclaimer:** Little Steve ColberT is wrong. Not only that, but he went to Northwestern, one of the best engineering schools in the country. He is intentionally misleading you.

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1I am America and So Can You p.120
2.1 Logs and Exponential Functions and Applications

\[
x = \log y \iff y = 10^x
\]

\[
x = \ln y \iff y = e^x
\]

More generally:

\[
x = \log_b y \iff y = b^x
\]

So \( \log_b(y) \) asks to what power must we raise \( b \) to get \( y \)?

**Most Basic Rules of Logarithms:** (Try proving these from the definition and exponent rules.)

- \( \log(x \cdot y) = \log(x) + \log(y) \)
- \( \log(x^p) = p \log(x) \)
- \( \log(1) = 0 \)

Either memorize these or know how to derive them from the definition or from the previous three rules:

- \( \log\left(\frac{1}{x}\right) = -\log(x) \)
- \( \log\left(\frac{x}{y}\right) = \log(x) - \log(y) \)
- \( \log(10^x) = x \)

- \( \log_b(b^x) = x \)
- \( \log(x + y) = \log(x + y) \) (Why did I include this?)

**Try these Problems:**

- Without a calculator, evaluate \( \log_2(4) \)
- Consider \( 7^{3x-2} = 5^{2x} \); solve for \( x \)
- \( \log(x) = 8, \log(y) = 3. \) Find \( \log\left(\frac{10^{x/3}}{y^{-5}}\right) \).
- Without a calculator, evaluate \( \ln(e), e^{\ln(x)}, \ln(e^4), \) and \( \ln(e^x) \).
Here is one last real life 34A Quiz.\(^2\) Try to answer it correctly on your own before you look at the solution.

\[ \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \]

\[ \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} \] Only when the denominator is the same!

\[ \frac{a}{b} = \frac{ac}{bc} \text{ when } c \neq 0 \text{ since } \frac{c}{c} = 1. \]

These can be used to derive the following rules.

\[ \frac{1}{\frac{a}{b}} = \frac{b}{a} \]

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\(^2\)A special thanks to one of Dr. Grigoryan’s 34A students for giving me permission to reproduce this image.