Consider the following system of two differential equations:
\[ \begin{align*}
  x'(t) &= x(t) - 2y(t) \\
  y'(t) &= -2x(t) + 4y(t)
\end{align*} \]
Let \( \hat{v}'(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} \) and \( \hat{v}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \).

1. The coefficient matrix is the matrix \( A \) so that \( \hat{v}'(t) = A\hat{v}(t) \). Write the coefficient matrix for this system. (You find this in exactly the same way as you would for a system of linear equations.)

2. Just like in calculus, there is no change when the derivative is zero! Use Gaussian elimination to solve \( A\hat{v}(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \)

Answer:
\[ x(t) = \underline{\phantom{0000}}, \quad y(t) = \underline{\phantom{0000}}\]
Or written another way:
\[ \hat{v}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = r \begin{pmatrix} \underline{\phantom{0000}} \\ \underline{\phantom{0000}} \end{pmatrix} \text{ where } r \text{ is } \underline{\phantom{0000}} .\]

This is the set of constant solutions to a system of differential equations!
**Instructions**: Work in groups of three to solve the systems of linear equations. Label the coefficient and augmented matrices as appropriate, and give your answer in vector form.

1.

\[
\begin{align*}
x - y + z &= 0 \\
x + 2y + z &= 6 \\
x - z &= 2
\end{align*}
\]

How many solutions are there?

2.

\[
\begin{align*}
2x_1 - x_2 + x_3 &= 0, \\
x_1 + 3x_2 + x_3 + 5x_4 &= 2, \\
-4x_1 + 2x_2 - 2x_3 &= 1
\end{align*}
\]

How many solutions are there?

3.

\[
\begin{align*}
x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 &= 0, \\
x_1 + 2x_2 + 2x_3 + 3x_4 + 4x_5 &= 0, \\
2x_1 + 4x_2 + 2x_3 + 4x_4 + 6x_5 &= 0
\end{align*}
\]

How many solutions are there?

**Check your understanding.**

1. Describe how you find solutions to a system of linear equations?

2. What are the different kinds of solutions one can have to a system of linear equations.

3. Why do you think we are beginning the course with linear algebra?