Variation of Parameters

Use clear presentation and justify every step.

1. What kinds of DE can be solved using variation of parameters?

2. What are the three steps of solving these DE’s using variation of parameters?

3. Solve the following DE using variation of parameters:

   \[ y'(t) + y(t) = t \]

   (a) Find the homogeneous solution \( y_h \).

   (b) Find a particular solution \( y_p \).

   (c) What is the general solution?

   Why is the last step valid? \(^1\)

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\(^1\)Hint: Substitute the sum into the left side and use the fact \( y_h \) is the solution to the homogeneous DE.
4. Solve the following differential equation using variation of parameters, which was outlined in the first problem. Indicate clearly every step of your solution.

\[ y'(t) - y(t) = \sin(t) \]

5. Suppose you win the CA Megamillions lottery, but you only get one million. You deposit this money in a bank account that pays an annual 9% interest compounded continuously and you withdraw 50,000$ over the course of each year.

(a) Set up an IVP for \( y(t) \), the amount of money in the bank \( t \) years after you first deposit it.

(b) Solve for \( y(t) \).

(c) How long will the money last?

(d) How long will the money last if you withdraw 900,000$ over each year?