1. What does it mean for two vectors to be linearly independent?

2. What does it mean for three vectors to be linearly independent?

3. If \( \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \), \( \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \), and \( \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \) are linearly independent\(^1\), then there do not exist nonzero constants \( x_1, x_2, x_3 \in \mathbb{R} \) so that

\[
x_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \hat{0}.
\]

(a) Write this as a system of linear equations.

(b) If these vectors are linearly independent, how many solutions will this system have and what will they be?

(c) Are these vectors linearly independent?

4. Are the vectors \( \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \) linearly independent?

\(^1\)This should help you with number 2.
5. What about the vectors \( \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \), \( \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \), and \( \begin{pmatrix} 5 \\ 10 \\ -1 \end{pmatrix} \)? Are they linearly independent?