Differential Equations Worksheet 1
Math 5A Winter 2010, TA Grace Kennedy

NAME: ________________________

Course Website: TBA
Section Website: http://math.ucsb.edu/~kgracekennedy/W105A.html

Supplemental Reading: Stewart Calculus Chapters 5 and Sections 7.1 and 7.4,
Farlow's DE and LA (your textbook) Chapters 1-3

REVIEW from 3B and C

No one should be left behind because they do not know or remember prerequisite
material. I’ve included some practice problems from 3B and 3C that you should
be able to do easily, if not today then by the end of the week.

1 Integration Methods, 3B

In addition to the integration “rules,” you must also recall several techniques of
integration.

1. \textit{u−substitution}

\begin{align*}
\text{a) } & \int x \sin(x^2 - 3) \, dx \\
\text{b) } & \int \frac{1}{1+x^2} \, dx \\
\text{c) } & \int \frac{1}{1-2y} \, dy
\end{align*}

\begin{align*}
u &= \\
\frac{du}{dx} &= \end{align*}

2. \textit{Integration by Parts}

\begin{align*}
\text{a) } & \int x \sin(x - 3) \, dx \\
\text{b) } & \int e^t \cos t \, dt \\
\text{c) } & \int \ln(y) \, dy
\end{align*}

\begin{align*}
u &= \\
\frac{du}{dx} &= \\
\frac{dv}{dy} &= \\
v &= \end{align*}
3. Partial Fractions

Find A and B to rewrite the expression:

\[ \frac{1}{x^2 - 4} = \frac{A}{x-2} + \frac{B}{x+2} \]

\[
\begin{align*}
a) \int \frac{1}{x^2 - 4} \, dx & \quad b) \int \frac{1}{x^2 + 5x + 6} \, dx \\
c) \int \frac{1}{1 - y^2} \, dy
\end{align*}
\]


2 Differential Equations, 3C

This course is a continuation of 3C, so you still need to know the material as well as you did the day of the exam, but here are a few calculations you should have down from day 1. You should also go back and review phase portraits, slope fields, and the linear algebra you did at the end of 3C. We won’t use the linear algebra immediately, but it will come back in Chapter 5.

5A is all about systems of differential equations that you did at the end of 3C, but to be able to solve these beasts, you must be able to solve plain-old differential equations as well.

For old 3C worksheets: http://math.ucsb.edu/~kgracekennedy/SB093C.html.

1. What is a differential equation (DE)?
2. What is a solution to a differential equation?
3. Are the following are solutions to the given differential equations:
   a) \( 9y'' + 4y = 0 \)
   \( y = 2 \cos \left( \frac{2}{3}t - \frac{\pi}{2} \right) \)
   b) \( y'' - 2y' + y = 0 \)
   \( y = 2e^{-t} - te^{-t} \)
   c) \( y'' = 25y \)
   \( y = 5e^{3t} \)

4. What does it mean for a DE to be linear? non-linear? homogeneous? non-homogeneous? What is the order of a differential equation?
5. What is a system of differential equations?
6. What is a solution to a system of differential equations?
7. Systems of Differential Equations

\[
\frac{dx}{dt} = 2 - x - y, \quad \frac{dy}{dt} = -y.
\]

Which of the following are solutions to this system of differential equations? Carefully show or explain why or why not.

a) \( x(t) = 0 \)
\( y(t) = 2 \)

b) \( x(t) = e^{-t}(1 - t) + 2 \)
\( y(t) = -e^{-t} \)

c) \( y(t) = e^{-t} \)

d) \( x(t) = e^{-t}(1 - t) + 2 \)
\( y(t) = e^{-t} \)

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d) Find all nullclines, equilibrium points, and give stability. Draw the phase portrait.\(^1\)

8. *Separation of Variables* First identify these DE’s as linear, non-linear, homogeneous, or non-homogeneous (all that apply) and give the order. Then give the general solution and solve the IVP of the following DE’s.

\begin{align*}
\text{a)} & \quad y' = 1 + y \quad & \text{b)} & \quad y' + y^2 - 9 = 0 \quad & \text{c)} & \quad ty' = 1 + y^2 \\
\quad y(0) = 0 & \quad & \quad y(0) = 0 & \quad & \quad y(1) = 1
\end{align*}

9. *Integrating Factor Method and Variation of Parameters* First identify these DE’s as linear, non-linear, homogeneous, or non-homogeneous (all that apply) and give the order. Give the general solution to the DE and solve the IVP if an initial value is given.

\begin{align*}
\text{a)} & \quad y' + y = sint \\
\quad y(0) = 1 & \quad & \text{b)} & \quad y' - y = e^{3t} \quad & \text{c)} & \quad y' + \frac{2t+1}{t-1}y = 2t
\end{align*}

\(^1\)You studied this type of question in 3C, but it has also come up on 5A final exams before.