Differential Equations Worksheet 4  
Math 5A Winter 2010, TA Grace Kennedy

NAME:  

Course Website: http://www.math.ucsb.edu/~yjshu/5aw10/  
Section Website: http://math.ucsb.edu/~kgracekennedy/W105A.html

Supplemental Reading: Farlow’s DE and LA (your textbook) Section 4.7

Converting Higher Order DE's To/From Systems of 1st Order DE's

Next time: Linear Transformations

Problem 1. On your homework, you were asked to make sense of the notation:

Assume \( \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} \)

\[ \mathbf{x}' = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \mathbf{x} + e^{t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

a) Turn this information into a system of differential equations. What would a solution look like?

Solving:

\[ x' = y \\ y' = 2x + te^t \]

How many DE's are there in your system? \( \text{[2]} \)

How would you classify each DE? (Give a list of vocabulary that describes the DE well enough that someone else might know how to solve it.)

1st order, linear, constant coefficients, nonhomogeneous

b) Now suppose this system of DE's represents a second order DE in \( z; z \) is a function of time. Give the DE that this system describes, and find its general solution.

\[ z = x \]

\[ z' = y \]

\[ z'' = y' = y + t e^t \text{ (in it does)} \]

\[ x' = y \Rightarrow z' = z' \checkmark \]

\[ y'' = 2x + te^t \Rightarrow z'' = 2z + te^t \]

\[ y = e^t \]

\[ z = z = e^t \]

DE: \( z'' - 2z = e^t \) - find \( y \) homogeneous (first order)

Guess: \( y_p = A e^t \)

\[ y_p'' = A e^t \]

\( \text{Sub into DE} \)

\[ y_p'' = A e^t \]

\[ A e^t - 2A e^t = e^t \]

\[ A e^t = e^t \]

\[ A = 1 \]

\[ y_p = -e^t \]