1. This is a solution to the first problem from the sheet of practice problems provided by Professor García-Cervera.

The augmented matrix for this system of linear equations is

\[
\begin{pmatrix}
1 & 2 & 3 & -5 \\
3 & 2 & 1 & -7 \\
0 & 5 & -1 & -32
\end{pmatrix}
\]

and row reduces to

\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & -6 \\
0 & 0 & 1 & 2
\end{pmatrix}.
\]

Translating this back into equations, we have \( x = 1, y = -6, z = 2 \). So in vector form, our solution set is

\[
\text{Solution Set} = \left\{ \begin{pmatrix} 1 \\ -6 \\ 2 \end{pmatrix} \right\}.
\]

Geometrically, our three equations each represented planes. When you look for \( x, y, \) and \( z \) that satisfy each of the equations simultaneously, you are looking for the set of points that lie in their intersection. Here, the three of the planes intersect simultaneously in exactly one point, the point in our solution set. (Try to think about how the \( xy-\), \( yz-\), and \( xz-\) planes intersect at exactly one point- the origin.)

2. This is a solution to the second problem from the sheet of practice problems provided by Professor García-Cervera.

The augmented matrix for this system of linear equations is

\[
\begin{pmatrix}
1 & -1 & 2 & 3 & 0 \\
-1 & 1 & 4 & 2 & 1
\end{pmatrix}
\]

and row reduces to

\[
\begin{pmatrix}
1 & -1 & 0 & 4 & 1 \\
0 & 0 & 1 & 3 & 2
\end{pmatrix}.
\]
Translating this back into equations, we have \( x - y + \frac{1}{3}w = \frac{2}{3} \) and \( z + \frac{5}{6}w = \frac{1}{6} \). The variables that are associated with our pivots are \( x \) and \( z \), so we put these variables in terms of the others, \( x = y - \frac{4}{3}w - \frac{1}{3} \) and \( z = -\frac{5}{6}w + \frac{1}{6} \). So in vector form, our solutions look like

\[
\begin{pmatrix}
x \\
y \\
z \\
w
\end{pmatrix} = \begin{pmatrix}
y - \frac{4}{3}w - \frac{1}{3} \\
y \\
-\frac{5}{6}w + \frac{1}{6} \\
w
\end{pmatrix} = \begin{pmatrix}
y \\
y \\
0 \\
w
\end{pmatrix} + \begin{pmatrix}
-\frac{4}{3}w \\
0 \\
-\frac{5}{6}w \\
w
\end{pmatrix} + \begin{pmatrix}
-\frac{1}{3} \\
0 \\
0 \\
\frac{1}{6}
\end{pmatrix}
\]

where \( y \) and \( w \) can be any real number.

We write the set of solutions

\[
\text{Solution Set} = \left\{ \begin{pmatrix}
r \\
1 \\
0 \\
0
\end{pmatrix} + s \begin{pmatrix}
-\frac{4}{3} \\
0 \\
-\frac{5}{6} \\
\frac{1}{6}
\end{pmatrix} \mid r, s \in \mathbb{R} \right\}
\]

Geometrically, our two equations each represent an affine subset of \( \mathbb{R}^4 \).

3. This is a solution to problem 2 on the back of Worksheet 1, which is posted on my website.

The augmented matrix for this system of linear equations is

\[
\begin{pmatrix}
2 & -1 & 1 & 0 & 0 \\
1 & 3 & 1 & 5 & 2 \\
-4 & 2 & -2 & 0 & 1
\end{pmatrix}
\]

and row reduces to

\[
\begin{pmatrix}
1 & 0 & \frac{4}{7} & \frac{5}{10} & 0 \\
0 & 1 & \frac{1}{7} & \frac{9}{10} & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\].

Translating this back into equations, we have \( x_1 + 0x_2 + \frac{4}{7}x_3 + \frac{5}{10}x_4 = 0 \), \( x_2 + \frac{1}{7}x_3 + \frac{9}{10}x_4 = 0 \), and \( 0x_1 + 0x_2 + 0x_3 + 0x_4 = 1 \). The last equation simplifies to \( 0 = 1 \), which is impossible. When we row reduce and realize our equations are equivalent to something that is impossible, we say the system is inconsistent, and there are no solutions.

Geometrically, the first and third equation are planes in \( \mathbb{R}^4 \). Dividing the third equation by negative two, we see that it is in fact ‘the same plane’ as the first equation, but it has been shifted up so that it does not pass through the origin.