Math 34B

Practice Exam, 3 hrs

March 15, 2012

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The net change of the volume of water in the reservoir is then
$\int_0^3 -90 - 2t^2\,dt$
$= (-90t - 2\frac{1}{3}t^3)|_0^3$
$= (-90 \cdot 3 - \frac{2}{3}3^3) - (-0 - 0)$
$= -270 - 18$
$= -252.$
Hence the volume of water leaving the reservoir is 252 acre-feet.

12.1.2 What are the amplitude, frequency, and period of the wave $11 + 8\sin(6t)$?

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Amplitude $A = 8/2 = 4$.
Period $T = \frac{2\pi}{6} = \pi/3$.
Frequency $f = 1/T = 3/\pi$.

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For min/max need to use the Second Derivative Test:
$y(x)'' = (e^x + e^{-x})'' = (e^x - e^{-x})' = e^x + e^{-x}$ evaluated at $x = 0$
$y''(0) = 1 + 1 = 2 > 0$
Hence there is only one critical point $x = 0$ which is a minimum

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The increase of $V(R)$ to the first order is $V'(R)\Delta R$.
Computing $V'(R) = (4\pi R^3/3)' = 4\pi R^2$.
The increase is $(4\pi R^2)\Delta R$.

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Integrate once more:

$y(t) = \int t^4 + t^2/2 + C\,dt = \frac{t^5}{5} + \frac{t^3}{6} + Ct + C_2.$

13.7.10 A lake of water is at temperature of 60. The air temperature drops to 30. Assume that Newton's law of cooling applies to the lake. If the temperature drops 10 during the first day, how long will it take till the temperature of the lake is 50?

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Initial temperature is $60 = 30 + Ae^{-k0} = 30 + A$, $A = 30$.
Drop during the first day is
$60 - 10 = 30 + 30e^{-k\cdot 1}$, $k = -\ln(20/30)$.
Temperature of the lake is $y = 30 + 30e^{-(-\ln 2/3)t} = 30 + 30e^{\ln(2/3)t}$.

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Temperature is 50 when $50 = 30 + 30e^{\ln(2/3)t}$, $20 = 30e^{\ln(2/3)t}$,
$2/3 = e^{\ln(2/3)t}$, $\ln 2/3 = \ln 2/3t$, $\boxed{t = 1}$.

15.0.04a What is the rate of increase with respect to x of the function $e^y(x^2 + 2x + 1)$?

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The rate of change is the derivative.
Need to calculate
$\frac{\partial}{\partial x} e^y(x^2 + 2x + 1) = e^y \frac{\partial}{\partial x}(x^2 + 2x + 1) = \boxed{e^y 2x + e^y 2}$.

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In our case $f_x = 3$, $f_y = -2$, $f(4, 3) = 12 - 6 = 6$.
Hence $z = 3(x - 4) - 2(y - 3) + 6 = 3x - 2y$.

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We have $x = 2$, $\Delta x = 0.003$, $y = 1$, $\Delta y = 0.04$,
$f_x(2, 1) = 6x|_{x=2} = 12$, $f_y(2, 1) = 2y|_{y=1} = 2$,
$f(2, 1) = 13$.

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$f(2, 1) = 13$.
So $f(2.003, 1.04) \approx 13 + 12 \cdot 0.003 + 2 \cdot 0.04 = \cdots$.

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At the minimum $f_x = f_y = 0$.
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$\boxed{x = 2/5,\ y = -8/5}$.

Teaching Evaluations

Please pick them up in doorway or on stage

Please fill out

(1) ESCI form: fill in two bubbles. Then pass LEFT

(2) Math Dept form. Write answers in dark pen to the two questions. Then pass RIGHT