Question Find all solutions to \( \ln x + \ln(x + 1) = 1 \)

Incorrect Start with \( \ln x + \ln(x + 1) = 1 \) and take exponent of both sides, we get
\[
e^{\ln x} \times e^{\ln(x+1)} = e^1
\]
Using the fact that \( e^{\ln x} = x \) we get
\[
x + x + 1 = 2x + 1 = e
\]
Now we can solve the linear equation to get \( x = \frac{e-1}{2} \).

Incorrect #2 Start with \( \ln x + \ln(x + 1) = 1 \). Take the exponent of both sides, we get
\[
e^x + e^{x+1} = e^x(1+1) = e \text{ then } e^x = \frac{e}{1+1}
\]
Next take the log of both sides to get \( x = \ln(e/2) \).

Miscellaneous Errors
- \( \ln e^x = 1 \rightarrow 1^e = x \).
- \( \ln x + \ln(x + 1) = \ln[x + (x + 1)] \)
- \( \ln x = 7 \rightarrow x = 7 \ln^{-1} \)

Correct Using the rule - \( \log a + \log b = \log ab \) - we multiply what’s inside the logs
\[
\ln x + \ln(x + 1) = \ln x(x + 1) = 1.
\]
Next we take the exponent of both sides to get rid of the logs
\[
x(x + 1) = x^2 + x = e \text{ or } x^2 + x - e = 0
\]
This is a quadratic equation. We can use the quadratic formula
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(-e)}}{2} = \frac{-1 \pm \sqrt{1 + 4e}}{2}
\]
However we reject the root with the - sign since we can’t take log of negative numbers. The only solution is \( \frac{-1 + \sqrt{1 + 4e}}{2} \).

Alternative We start with \( \ln x + \ln(x + 1) = 1 \) and take exponents of both sides
\[
e^{\ln x} \times e^{\ln(x+1)} = e^1
\]
Now we remember that \( e^{\ln x} = x \) so
\[
x(x + 1) = x^2 + x = e \text{ or } x^2 + x - e
\]
and we use the quadratic formula.