High Dimensional Expanders

Luis Kumanduri

MIT
What is an expander?

**Definition**
Let $X$ be a $d$-dimensional simplicial complex. $X$ is an $\epsilon$-topological expander if for every continuous $F : X \rightarrow \mathbb{R}^d$, there is a point $p \in \mathbb{R}^d$ so that $F^{-1}(p)$ meets an $\epsilon$ fraction of the $d$-dimensional faces of $X$.

**Theorem**
*(Gromov)* If $X$ has large cosystoles, satisfies a co-isoperimetric inequality and is sparse, then $X$ is a topological expander.
Questions

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Can we develop better tests for expansion? In particular, what topological/geometric properties does expansion imply?

Question

Can we algorithmically estimate the expansion constant for a given complex? Somewhat relatedly, can we improve the bounds on the constant in Gromov’s theorem?