WeBWorK assignment number Week1 is due: 01/15/2014 at 04:00am PST.

The (* home page * )
for the course contains the syllabus, grading policy and other information.

This file is /conf/snippets/setHeader.pg you can use it as a model for creating files which introduce each problem set.

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Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 2^3 instead of 8, sin(3 * pi/2) instead of -1, e / (ln(2)) instead of 2, (2 + tan(3)) * (4 - sin(5)) / 6 - 7/8 instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

1. (1 pt) Write the augmented matrix of the system

\[
\begin{bmatrix}
x + 51y - 15z &= -16 \\
-44y - 6z &= 0 \\
-79x + 74z &= -9 \\
\end{bmatrix}
\]

Find the reduced row echelon form of this augmented matrix.

2. (1 pt)

Use elementary row operations to find the reduced row echelon form of

\[
\begin{bmatrix}
3 & 0 & 0 \\
8 & 16 & 24 \\
10 & 8 & 12 \\
0 & 24 & 36 \\
\end{bmatrix}
\]

Which columns are pivot columns? Enter the sum of the column numbers. For instance if the first and third columns were pivot columns then your answer would be 1+3=4.

3. (1 pt)

Consider the system

\[
\begin{align*}
x - y - 2z &= 15 \\
2x + 3y + z &= 10 \\
5x + 4y + 2z &= 6 \\
\end{align*}
\]

What is the augmented matrix of the sytem?

Find the reduced row echelon form of this augmented matrix.

4. (1 pt)

Consider the system

\[
\begin{align*}
4x_1 + 4x_2 + 8x_3 &= 0 \\
2x_1 - x_2 + x_3 &= 0 \\
14x_1 - 4x_2 + 10x_3 &= 0 \\
\end{align*}
\]

What is the augmented matrix of the sytem?

Find the reduced row echelon form of this augmented matrix.

5. (1 pt)

Determine any values of k which make the augmented matrix

\[
\begin{bmatrix}
1 & 2 & -6 & 1 \\
3 & 4 & 3 & k \\
\end{bmatrix}
\]

inconsistent.

Enter your answers in increasing order. If less than 3 values of k make it inconsistent, enter X into any blanks you wouldn’t
use. If all values of k make it inconsistent enter A into all 3 blanks.

6. (1 pt)
Consider the matrix
\[
\begin{bmatrix}
1 & 3 & 7 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
Which condition or conditions of being in RREF does the matrix fail?
- A. Zero rows are at the bottom
- B. The leftmost nonzero entry of each nonzero row equals 1
- C. Each pivot is further to the right than the pivot in the row above it
- D. Each pivot is the only nonzero entry in its column
- E. None of the above, it is in RREF

7. (1 pt)
Consider the matrix
\[
\begin{bmatrix}
1 & 10 & 1 & 5 \\
0 & 1 & 5 & 9 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
Which condition or conditions of being in RREF does the matrix fail?
- A. Zero rows are at the bottom
- B. The leftmost nonzero entry of each nonzero row equals 1
- C. Each pivot is further to the right than the pivot in the row above it
- D. Each pivot is the only nonzero entry in its column
- E. None of the above, it is in RREF

8. (1 pt)
Consider the matrix
\[
\begin{bmatrix}
1 & 6 & 0 & 3 & 0 \\
0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
Which condition or conditions of being in RREF does the matrix fail?
- A. Zero rows are at the bottom
- B. The leftmost nonzero entry of each nonzero row equals 1
- C. Each pivot is further to the right than the pivot in the row above it
- D. Each pivot is the only nonzero entry in its column
- E. None of the above, it is in RREF

9. (1 pt)
Use elementary row operations to find the reduced row echelon form of
\[
\begin{bmatrix}
1 & 3 & 8 & 0 \\
5 & 16 & 42 & 1 \\
0 & 7 & 14 & 28
\end{bmatrix}
\]
Which columns are pivot columns? Enter the sum of the column numbers. For instance if the first and third columns were pivot columns then your answer would be 1+3=4.

10. (1 pt) Solve the system using substitution
\[
\begin{cases}
x - 2y = -4 \\
4x - 6y = -5
\end{cases}
\]
x = 
y = 

11. (1 pt) Solve the system using matrices (row operations)
\[
\begin{bmatrix}
-8x - 7y = 104 \\
3x - 7y = 38
\end{bmatrix}
\]
x = 
y = 

12. (1 pt) Solve the system by using Cramer’s Rule.
\[
\begin{bmatrix}
2x - 9y = 82 \\
3x + 7y = -82
\end{bmatrix}
\]
x = 
y = 

13. (1 pt) Solve the system using matrices (row operations)
\[
\begin{bmatrix}
4x - 3y + 3z = 11 \\
5x + 2y + 2z = -18 \\
6x + 5y - 4z = -64
\end{bmatrix}
\]
x = 
y = 
z = 

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1. (1 pt) Determine the value of $h$ such that the matrix is the augmented matrix of a consistent linear system.

\[
\begin{bmatrix}
8 & -2 & h \\
-24 & 6 & 4
\end{bmatrix}
\]

$h =$

2. (1 pt) Determine the value of $h$ such that the matrix is the augmented matrix of a linear system with infinitely many solutions.

\[
\begin{bmatrix}
8 & -6 & 4 \\
32 & h & 16
\end{bmatrix}
\]

$h =$

3. (1 pt) Solve the equation

\[8x + 7y + 9z = -15.\]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} s + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t.
\]

4. (1 pt) Determine the value of $k$ for which the system

\[
\begin{cases}
x + y + 2z = 2 \\
x + 2y - 5z = 2 \\
2x + 6y + kz = 5
\end{cases}
\]

has no solutions.

$k =$

5. (1 pt) Solve the system

\[
\begin{cases}
-4x_1 + x_2 = 3 \\
8x_1 - 2x_2 = -6
\end{cases}
\]

\[
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} s.
\]

6. (1 pt) Solve the system

\[
\begin{cases}
x_1 + x_2 + 3x_3 = -4 \\
6x_1 + 5x_2 + 3x_3 = 5
\end{cases}
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix} s.
\]

7. (1 pt) Solve the system

\[
\begin{cases}
x_1 - x_2 - 2x_3 = -6 \\
3x_1 - 4x_2 - 3x_3 = -9 \\
3x_1 - 15x_2 = -45
\end{cases}
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} s.
\]

8. (1 pt) Solve the system

\[
\begin{cases}
x_1 + 3x_2 + 2x_3 = 10 \\
x_2 - 2x_3 - 3x_4 = -4 \\
3x_1 - 2x_2 + 15x_3 + 12x_4 = 46 \\
-x_2 + 2x_3 + 8x_4 = -6
\end{cases}
\]

\[
x_1 = \\
x_2 = \\
x_3 = \\
x_4 = 
\]

9. (1 pt) Solve the system

\[
\begin{cases}
x_1 + x_2 = 4 \\
x_2 + x_3 = 5 \\
x_1 + x_3 + x_4 = -5 \\
x_1 + x_4 = -6
\end{cases}
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \\ 1 \end{bmatrix} s.
\]
10. (1 pt) Solve the system

\[
\begin{align*}
5x_1 - 4x_2 + 4x_3 + 2x_4 &= 3 \\
-x_1 + x_2 + 3x_3 + 3x_4 &= 5 \\
4x_1 - 3x_2 + 7x_3 + 5x_4 &= 8 \\
2x_1 - 2x_2 - 6x_3 - 6x_4 &= -10
\end{align*}
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} = \begin{bmatrix}
- & - & - & - \\
- & - & - & - \\
- & - & - & - \\
- & - & - & - \\
\end{bmatrix} s + \begin{bmatrix}
- \\
- \\
- \\
- \\
\end{bmatrix} t.
\]
1. (1 pt)
For each system, determine whether it has a unique solution (in this case, find the solution), infinitely many solutions, or no solutions.

1. \[
\begin{align*}
4x+4y &= 12 \\
-4x-4y &= -11
\end{align*}
\]
- A. Unique solution: \(x = 12, y = -11\)
- B. No solutions
- C. Unique solution: \(x = -11, y = 12\)
- D. Unique solution: \(x = 0, y = 0\)
- E. Infinitely many solutions
- F. None of the above

2. \[
\begin{align*}
5x-5y &= 0 \\
8x-4y &= 0
\end{align*}
\]
- A. Unique solution: \(x = 4, y = 5\)
- B. Infinitely many solutions
- C. Unique solution: \(x = 0, y = 4\)
- D. Unique solution: \(x = 0, y = 0\)
- E. No solutions
- F. None of the above

3. \[
\begin{align*}
-9x+4y &= -42 \\
-2x+5y &= -34
\end{align*}
\]
- A. Unique solution: \(x = 2, y = -6\)
- B. Unique solution: \(x = -6, y = 2\)
- C. Unique solution: \(x = 0, y = 0\)
- D. No solutions
- E. Infinitely many solutions
- F. None of the above

4. \[
\begin{align*}
-2x+2y &= 6 \\
-6x+6y &= 18
\end{align*}
\]
- A. Unique solution: \(x = 6, y = 18\)
- B. No solutions
- C. Unique solution: \(x = -3, y = 0\)
- D. Infinitely many solutions
- E. Unique solution: \(x = 0, y = 0\)

2. (1 pt) The reduced row-echelon forms of the augmented matrices of four systems are given below. How many solutions does each system have?

1. \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 5 \\
0 & 0 & 0
\end{bmatrix}
\]
- A. No solutions
- B. Unique solution
- C. Infinitely many solutions
- D. None of the above

2. \[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 0 & 1 & -9
\end{bmatrix}
\]
- A. No solutions
- B. Unique solution
- C. Infinitely many solutions
- D. None of the above

3. \[
\begin{bmatrix}
0 & 1 & 0 & -14 \\
0 & 0 & 1 & 15
\end{bmatrix}
\]
- A. Unique solution
- B. Infinitely many solutions
- C. No solutions
- D. None of the above

4. \[
\begin{bmatrix}
1 & 0 & -2 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
- A. Unique solution
- B. No solutions
- C. Infinitely many solutions
- D. None of the above

3. (1 pt) Write the system
\[
\begin{align*}
4y+9z &= -2 \\
2x-6y &= 11 \\
3x-7y+10z &= -3
\end{align*}
\]
in matrix form.
\[
\begin{bmatrix}
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
= 
\begin{bmatrix}
  \_ \\
  \_ \\
  \_ \\
\end{bmatrix}
\]

4. (1 pt) 
\(-2x + 7y - 3z = -7\)
\(2x - 4y + 5z = 4\)
\(-3x + 5y - 2z = -5\)
Write the above system of equations in matrix form:
\[
\begin{bmatrix}
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
= 
\begin{bmatrix}
  \_ \\
  \_ \\
  \_ \\
\end{bmatrix}
\]

5. (1 pt)
Write the system of equations in matrix-vector form, then write the augmented matrix of the system.
\(-2x + 7y - 3z = -7\)
\(2x - 4y + 5z = 4\)
\(-3x + 5y - 2z = -5\)
\[
\begin{bmatrix}
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
= 
\begin{bmatrix}
  \_ \\
  \_ \\
  \_ \\
\end{bmatrix}
\]

6. (1 pt)
Write the system of equations in matrix-vector form, then write the augmented matrix of the system.
\(-2x + 7y - 3z = -7\)
\(2x - 4y + 5z = 4\)
\(-3x + 5y - 2z = -5\)
\[
\begin{bmatrix}
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
  \_ & \_ & \_ \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
= 
\begin{bmatrix}
  \_ \\
  \_ \\
  \_ \\
\end{bmatrix}
\]

7. (1 pt)
The system
\(10x + 3y = 6\)
\(40x + 12y = 6\)
of two linear equations in two variables represents which of the following cases?
- A. intersecting lines
- B. parallel lines
- C. a single line

8. (1 pt)
The system
\(3x - 8y = 1\)
\(-24x + 64y = -7\)
of two linear equations in two variables represents which of the following cases?
- A. intersecting lines
- B. parallel lines
- C. a single line

9. (1 pt)
\(V = \mathbb{R}^2\)
\(S = \{(0, 0), (-8, 10)\}\)
Do the vectors in the set \(S\) span the vector space \(V\)?
- A. Yes
- B. No

10. (1 pt)
\(V = \mathbb{R}^3\)
\(S = \{(1, 0, 0), (0, 1, 0), (-8, 7, 4)\}\)
Do the vectors in the set \(S\) span the vector space \(V\)?
- A. Yes
- B. No

11. (1 pt)
\(V = \mathbb{R}^3\)
\(S = \{(1, 0, -1), (9, 0, 6), (-6, 0, 1), (7, 0, 4)\}\)
Do the vectors in the set \(S\) span the vector space \(V\)?
- A. Yes
- B. No

12. (1 pt) Steve and Kate are brother and sister. Steve has twice as many sisters as brothers, and Kate has as many brothers as sisters. How many boys and girls are there in this family?
Answer: _____ boys and _____ girls.

13. (1 pt) Find the quadratic polynomial whose graph goes through the points \((-2, 18), (0, 6), \) and \((1, 12)\).
\(f(x) = _____ x^2 + _____ x + _____\)
14. (1 pt) Consider the chemical reaction
\[ aCH_4 + bO_2 \rightarrow cCO_2 + dH_2O, \]
where \( a, b, c, \) and \( d \) are unknown positive integers. The reaction must be balanced; that is, the number of atoms of each element must be the same before and after the reaction. For example, because the number of oxygen atoms must remain the same,
\[ 2b = 2c + d. \]
While there are many possible choices for \( a, b, c, \) and \( d \) that balance the reaction, it is customary to use the smallest possible integers. Balance this reaction.
\[ a = \quad b = \quad c = \quad d = \]

15. (1 pt) A dietician is planning a meal that supplies certain quantities of vitamin C, calcium, and magnesium. Three foods will be used, their quantities measured in milligrams. The nutrients supplied by one unit of each food and the dietary requirements are given in the table below.

<table>
<thead>
<tr>
<th>Nutrient</th>
<th>Food 1</th>
<th>Food 2</th>
<th>Food 3</th>
<th>Total Required (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vitamin C</td>
<td>50</td>
<td>25</td>
<td>75</td>
<td>587.5</td>
</tr>
<tr>
<td>Calcium</td>
<td>40</td>
<td>30</td>
<td>80</td>
<td>595</td>
</tr>
<tr>
<td>Magnesium</td>
<td>20</td>
<td>30</td>
<td>50</td>
<td>385</td>
</tr>
</tbody>
</table>

Write the augmented matrix for this problem.
\[
\begin{bmatrix}
  - & - & - & - \\
  - & - & - & - \\
  - & - & - & -
\end{bmatrix}
\]

What quantity (in units) of Food 1 is necessary to meet the dietary requirements?
___

What quantity (in units) of Food 2 is necessary to meet the dietary requirements?
___

What quantity (in units) of Food 3 is necessary to meet the dietary requirements?
___
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1. (1 pt) Find $a$ and $b$ such that

$$\begin{bmatrix} 31 \\ 21 \\ -1 \end{bmatrix} = a \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix} + b \begin{bmatrix} 8 \\ 6 \\ -1 \end{bmatrix}.$$  

$a =$ ______ 

$b =$ ______

2. (1 pt) Compute the following:

$$\begin{bmatrix} 9 & -8 & -5 \\ -8 & 5 & 1 \\ 5 & 5 & 5 \end{bmatrix} + \begin{bmatrix} 9 & -4 & 8 \\ -1 & 4 & 1 \\ -6 & 2 & 2 \end{bmatrix} = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$$

3. (1 pt) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 4 & 2 \\ -3 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 4 & -4 \\ 2 & -4 & -4 \\ 4 & -3 & 2 \end{bmatrix}$

Then $4A + B = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$

and $A^{T} = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$

4. (1 pt) If $A = \begin{bmatrix} -3 & 4 & -4 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 & -3 \\ 0 & 4 & 4 \\ -4 & 3 & -3 \end{bmatrix}$

Then $4A - 4B = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$

and $2A^{T} = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$

5. (1 pt) If $A = \begin{bmatrix} 1 & -3 & -2 \\ 0 & 4 & 4 \\ 4 & -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & -4 & 2 \\ 1 & -4 & 0 \\ -4 & 1 & 4 \end{bmatrix}$

Then $AB = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$

and $BA = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$

6. (1 pt) Compute the following product.

$$\begin{bmatrix} 0 & 1 \\ -1 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} & \phantom{-} \end{bmatrix}$$

7. (1 pt) If $A = \begin{bmatrix} 3 & 2 & 10 \\ -14 & 7 & 6 \\ 19 & -5 & 2 \\ 9 \end{bmatrix}$, determine the following entries:

$a_{13} =$ ______

$a_{21} =$ ______

$a_{32} =$ ______

8. (1 pt) If $A = \begin{bmatrix} 9 & -8 \\ 9 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} -10 & 7 \\ 4 & 7 \end{bmatrix}$

Then $AB = \begin{bmatrix} \phantom{-} & \phantom{-} \\ \phantom{-} & \phantom{-} \end{bmatrix}$

9. (1 pt) If $A$, $B$, and $C$ are $2 \times 2$, $2 \times 7$, and $7 \times 8$ matrices respectively, determine which of the following products are defined. For those defined, enter the size of the resulting matrix (e.g. "3 x 4", with spaces between numbers and "x"). For those undefined, enter "undefined".

$AC$: ______

$BC$: ______

$BA$: ______

$AB$: ______

10. (1 pt)

$V = \mathbb{R}^{2}$

$S = \{(1,7),(1,-7)\}$

Are the vectors in the set $S$ linearly independent in the vector space $V$?

- A. Yes
- B. No

!"
11. (1 pt)
\[ V = \mathbb{R}^3 \]
\[ S = \{(3, 3, -1), (-3, -5, -4), (2, -5, -3)\} \]

Are the vectors in the set \( S \) linearly independent in the vector space \( V \)?

- A. Yes
- B. No
WeBWorK assignment number setWeek3-3 is due: 01/24/2014 at 04:00am PST.

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1. (1 pt) Compute the following:
$$\begin{bmatrix} 6 & 6 & -4 \\ 9 & -1 & 4 \\ 2 & 4 & 8 \end{bmatrix} \ast \begin{bmatrix} 2 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

2. (1 pt) Compute the following product.
$$\begin{bmatrix} 7 & 5 \\ -8 & -9 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} -8 \\ 7 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

3. (1 pt) Compute the following product.
$$\begin{bmatrix} -2 & -3 & 3 \\ 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

4. (1 pt) Find a $3 \times 3$ matrix $A$ such that
$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 0 \end{bmatrix}, \quad A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}, \quad A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}.$$
1. (1 pt) Laying the Linearity on the Line

Let \( T(\begin{bmatrix} x \\ y \end{bmatrix}) = Ax \) where \( A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \)

In terms of \( x \) and \( y \), what does \( T(\begin{bmatrix} 0 \\ y \end{bmatrix}) \) equal?

\[ \_ \_ \_ \_ \_ \]

2. (1 pt) Geometric Interpretations in \( \mathbb{R}^2 \)

Let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \), \( T(x,y) = (x,-y) \).

(a) Construct a matrix interpretation of \( T \): \( \_ \_ \_ \_ \_ \_ \_ \_ \)

(b) Which of the following best describes the geometric interpretation of \( T \)?

- A. Reflection about the x-axis
- B. Reflection about the y-axis
- C. Projection onto the x-axis
- D. Projection onto the y-axis
- E. Projection onto the 45 degree line \( y=x \)

3. (1 pt) Geometric Interpretations in \( \mathbb{R}^2 \)

Let \( T : \mathbb{R}^2 \to \mathbb{R}^2 \), \( T(x,y) = (x,x) \).

(a) Construct a matrix interpretation of \( T \): \( \_ \_ \_ \_ \_ \_ \_ \_ \)

(b) Which of the following best describes the geometric interpretation of \( T \)?

- A. Reflection about the x-axis
- B. Reflection about the y-axis
- C. Projection onto the x-axis
- D. Projection onto the y-axis
- E. Projection onto the 45 degree line \( y=x \)

4. (1 pt) Find the Standard Matrix

Let \( T(x,y) = (y,-x) \)

Determine the standard matrix \( A \) such that \( T(v) = Av \).

\[ \_ \_ \_ \_ \_ \]

5. (1 pt) Find the Standard Matrix

Let \( T(x,y) = (x+2y,x-2y,y) \)

Determine the standard matrix \( A \) such that \( T(v) = Av \).

\[ \_ \_ \_ \_ \_ \]

6. (1 pt) Given:

\[ T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \]

\[ T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} -6 \\ 0 \end{bmatrix} \]

\[ T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} -5 \\ 1 \end{bmatrix} \]

Find a matrix such that:

\( T(\vec{v}) = \begin{bmatrix} _-_ & _-_ & _- \end{bmatrix} (\vec{v}) \)

7. (1 pt) Given:

\[ T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} -1 \\ -5 \end{bmatrix} \]

\[ T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 6 \\ -7 \end{bmatrix} \]

Find a matrix such that:

\( T(\vec{v}) = \begin{bmatrix} _-_ & _-_ & _- \end{bmatrix} (\vec{v}) \)
8. (1 pt) Given:

\[ T\left( \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 9 \\ -14 \end{bmatrix} \]

\[ T\left( \begin{bmatrix} -4 \\ -3 \end{bmatrix} \right) = \begin{bmatrix} -23 \\ 28 \end{bmatrix} \]

Find a matrix such that:

\[ T(\vec{v}) = \begin{bmatrix} \_ \\ \_ \end{bmatrix} (\vec{v}) \]
WeBWorK assignment number setWeek4-1 is due: 02/05/2014 at 06:00 am PST.

The Gauchospace link for the course contains the syllabus, grading policy and other information.

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Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2^3$ instead of 8, $\sin(3 \pi / 2)$ instead of -1, $e^{\ln(2)}$ instead of 2, $(2 + \tan(3)) \ast (4 - \sin(5)) \ast 6 - 7 / 8$ instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

1. (1 pt)
If the computation below cannot be done, type X in every box.

If $C = \begin{bmatrix} 4 & 1 \\ 2 & 4 \\ 0 & -4 \end{bmatrix}$ and $D = \begin{bmatrix} -3 & -2 & 0 \\ -1 & 3 & -3 \end{bmatrix}$

Then $CD = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

1!

2. (1 pt)
If the computation below cannot be done, type X in every box.

If $C = \begin{bmatrix} -4 & -1 \\ 4 & -3 \\ -1 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & -4 & 2 \\ -4 & 2 & -4 \end{bmatrix}$

Then $(DC)^T = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

1!

3. (1 pt)
If the computation below cannot be done, type X in every box.

If $A = \begin{bmatrix} -3 & -4 & 0 \\ 4 & 4 & 4 \\ 4 & 2 & -1 \end{bmatrix}$

Then $A^2 = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

1!

4. (1 pt)
If the computation below cannot be done, type X in every box.

If $B = \begin{bmatrix} 1 & -1 & -3 \\ -2 & -2 & 3 \\ 1 & 0 & -1 \end{bmatrix}$

Then $4B - 3I_3 = \begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

1!

5. (1 pt)
Multiply the given matrices.

$\begin{bmatrix} 6 & -8 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1/6 \\ -1/8 & 1/8 \end{bmatrix}$

Is one the inverse of the other?

- A. Yes
- B. No

1!

6. (1 pt)
Use row reduction to calculate the inverse of the given matrix.

$\begin{bmatrix} 9 & 6 \\ 4 & 2 \end{bmatrix}$

1!

7. (1 pt)
The inverse of $\begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$ is $\begin{bmatrix} 3/8 & -1/4 \\ -1/4 & 1/2 \end{bmatrix}$

Use this information to solve the system:

$\begin{align*}
4x_1 + 2x_2 &= -2 \\
2x_1 + 3x_2 &= 3
\end{align*}$

$x_1 =$

$x_2 =$

1!
8. (1 pt)
Find the inverse of
\[
\begin{bmatrix}
0 & 1 & 1 \\
3 & 1 & -1 \\
7 & -7 & -7
\end{bmatrix}
\]

Use this information to solve the system:
\[
\begin{align*}
y + z &= 6 \\
3x + y - z &= 4 \\
7x - 7y - 7z &= 0
\end{align*}
\]

\begin{align*}
x &= \\
y &= \\
z &=
\end{align*}
WeBWorK assignment number setWeek4-1new is due: 02/07/2014 at 04:00am PST.

The Gauchospace link for the course contains the syllabus, grading policy and other information.

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Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2^3$ instead of 8, $\sin(3 \times \pi/2)$ instead of -1, $e^{\ln(2)}$ instead of 2, $(2 + \tan(3)) \times (4 - \sin(5)) \times 6 - 7/8$ instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

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1. (1 pt) Multiply the given matrices.
   \[
   \begin{bmatrix}
   5 & -8 \\
   5 & 0
   \end{bmatrix}
   \begin{bmatrix}
   0 & 1/5 \\
   -1/8 & 1/8
   \end{bmatrix}
   \]
   Is one the inverse of the other?
   - A. Yes
   - B. No

2. (1 pt) Use row reduction to calculate the inverse of the given matrix
   \[
   \begin{bmatrix}
   5 & 1 \\
   3 & 2
   \end{bmatrix}
   \]
   ¡!

3. (1 pt) Are the following matrices invertible? Enter "Y" or "N". You must get all of the answers correct to receive credit.
   -1. \[
   \begin{bmatrix}
   5 & -9 \\
   -10 & 18
   \end{bmatrix}
   \]
   -2. \[
   \begin{bmatrix}
   3 & -7 \\
   -7 & -3
   \end{bmatrix}
   \]
   -3. \[
   \begin{bmatrix}
   -10 & -9 \\
   0 & 0
   \end{bmatrix}
   \]
   -4. \[
   \begin{bmatrix}
   7 & 5 \\
   4 & -7
   \end{bmatrix}
   \]

4. (1 pt) If \(A = \begin{bmatrix} -9 & -6 \\ 3 & -3 \end{bmatrix}\)
   Then \(A^{-1} = \begin{bmatrix} \text{ } & \text{ } \\ \text{ } & \text{ } \end{bmatrix}\)

5. (1 pt) The matrix \( \begin{bmatrix} 2 & -3 \\ -1 & k \end{bmatrix} \) is invertible if and only if \( k \neq \ldots \).

6. (1 pt) If \( A = \begin{bmatrix} 1 & -4 & -2 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \)
   Then \( A^{-1} = \begin{bmatrix} \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } \end{bmatrix} \)

7. (1 pt) If \( A = \begin{bmatrix} 5 & -5 & -29 \\ -5 & 6 & 33 \\ -1 & 1 & 6 \end{bmatrix} \)
   Then \( A^{-1} = \begin{bmatrix} \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } \end{bmatrix} \)

8. (1 pt) If \( A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ -6 & 1 & 1 & 0 \\ 2 & -2 & -1 & 1 \end{bmatrix} \)
   Then \( A^{-1} = \begin{bmatrix} \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \end{bmatrix} \)

9. (1 pt) Determine which of the formulas hold for all invertible \( n \times n \) matrices \( A \) and \( B \)
   - A. \((A + B)^2 = A^2 + B^2 + 2AB\)
   - B. \((I_n - A)(I_n + A) = I_n - A^2\)
   - C. \(AB = BA\)
   - D. \((AB)^{-1} = A^{-1}B^{-1}\)
   - E. \(3A\) is invertible
   - F. \(A + I_n\) is invertible
10. (1 pt)
The inverse of \[
\begin{bmatrix}
2 & 2 \\
2 & 4
\end{bmatrix}
\]
is \[
\begin{bmatrix}
1 & -1/2 \\
-1/2 & 1/2
\end{bmatrix}
\]
Use this information to solve the system:
\[
\begin{align*}
2x_1 + 2x_2 &= -2 \\
2x_1 + 4x_2 &= 3
\end{align*}
\]
\[
x_1 = \_\_\_
\]
\[
x_2 = \_\_\_
\]

11. (1 pt)
Find the inverse of \[
\begin{bmatrix}
0 & 1 & 1 \\
6 & 1 & -1 \\
8 & -8 & -8
\end{bmatrix}
\]
Use this information to solve the system:
\[
\begin{align*}
y + z &= 1 \\
6x + y - z &= 2 \\
8x - 8y - 8z &= 0
\end{align*}
\]
\[
x = \_\_\_
\]
\[
y = \_\_\_
\]
\[
z = \_\_\_
\]
WeBWorK assignment number setWeek5-1 is due: 02/12/2014 at 04:00am PST.

The
Gauchospace link
for the course contains the syllabus, grading policy and other information.

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Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2^3$ instead of 8, $\sin(3*\pi/2)$ instead of -1, $e^{ln(2)}$ instead of 2, $(2 + \tan(3)) * (4 - \sin(5)) / 6 - 7/8$ instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

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1. (1 pt) Find the determinant of the matrix

$$C = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & -2 \\ -2 & -1 & 1 & 0 \\ -2 & 2 & 3 & 1 \end{vmatrix}.$$  

$$\det(C) = \ldots .$$

2. (1 pt) If $A$ and $B$ are $2 \times 2$ matrices, $\det(A) = 2$, $\det(B) = 9$, then 

$$\det(AB) = \ldots ,$$  

$$\det(3A) = \ldots ,$$  

$$\det(A^T) = \ldots ,$$  

$$\det(B^{-1}) = \ldots ,$$  

$$\det(B^3) = \ldots .$$

3. (1 pt) Find the determinant of the matrix

$$M = \begin{vmatrix} -2 & -2 & 7 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{vmatrix}.$$  

$$\det(M) = \ldots .$$

4. (1 pt) If the determinant of a $5 \times 5$ matrix $A$ is $\det(A) = 2$, and the matrix $B$ is obtained from $A$ by multiplying the third row by 6, then $\det(B) = \ldots .$

5. (1 pt) If $\det\begin{vmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{vmatrix} = 1$ and $\det\begin{vmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{vmatrix} = -2,$

then $\det\begin{vmatrix} a & 6 & d \\ b & 6 & e \\ c & 6 & f \end{vmatrix} = \ldots ,$

and $\det\begin{vmatrix} a & -6 & d \\ b & -9 & e \\ c & -12 & f \end{vmatrix} = \ldots .$

6. (1 pt) If a $4 \times 4$ matrix $A$ with rows $v_1$, $v_2$, $v_3$, and $v_4$ has determinant $\det(A) = 8$,

then $\det\begin{vmatrix} v_1 \\ 5v_1 + 2v_2 \\ 6v_1 + 7v_3 \\ v_4 \end{vmatrix} = \ldots.$

7. (1 pt) Find the area of the parallelogram defined by the vectors

$$\begin{bmatrix} 4 \\ 7 \end{bmatrix}$$  

and  

$$\begin{bmatrix} -6 \\ 9 \end{bmatrix}.$$  

Area = \ldots .

8. (1 pt) Find the volume of the parallelepiped defined by the vectors

$$\begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \text{ and } \begin{bmatrix} 2 \\ -4 \end{bmatrix}.$$  

Volume = \ldots .

9. (1 pt) Does the following set of vectors constitute a vector space? Assume "standard" definitions of the operations.

The set of vectors in the first quadrant of the plane.

• A. Yes
• B. No

If not, which condition(s) below does it fail? (Check all that apply)

• A. Vector spaces must be closed under addition
• B. Vector spaces must be closed under scalar multiplication
• C. There must be a zero vector
• D. Every vector must have an additive inverse
• E. Addition must be associative
• F. Addition must be commutative
• G. Scalar multiplication by 1 is the identity operation
• H. The distributive property
• I. Scalar multiplication must be associative
• J. None of the above, it is a vector space

¡!
10. (1 pt)

Does the following set of vectors constitute a vector space? Assume "standard" definitions of the operations.

The set of all diagonal $2 \times 2$ matrices.

- A. Yes
- B. No

If not, which condition(s) below does it fail? (Check all that apply)

- A. Vector spaces must be closed under addition
- B. Vector spaces must be closed under scalar multiplication
- C. There must be a zero vector
- D. Every vector must have an additive inverse
- E. Addition must be associative
- F. Addition must be commutative
- G. Scalar multiplication by 1 is the identity operation
- H. The distributive property
- I. Scalar multiplication must be associative
- J. None of the above, it is a vector space

11. (1 pt)

Does the following set of vectors constitute a vector space? Assume "standard" definitions of the operations.

The set of all $2 \times 2$ matrices with determinant equal to zero.

- A. Yes
- B. No

If not, which condition(s) below does it fail? (Check all that apply)

- A. Vector spaces must be closed under addition
- B. Vector spaces must be closed under scalar multiplication
- C. There must be a zero vector
- D. Every vector must have an additive inverse
- E. Addition must be associative
- F. Addition must be commutative
- G. Scalar multiplication by 1 is the identity operation
- H. The distributive property
- I. Scalar multiplication must be associative
- J. None of the above, it is a vector space

12. (1 pt)

Does the following set of vectors constitute a vector space? Assume "standard" definitions of the operations.

The set of all invertible $2 \times 2$ matrices.

- A. Yes
- B. No

If not, which condition(s) below does it fail? (Check all that apply)

- A. Vector spaces must be closed under addition
- B. Vector spaces must be closed under scalar multiplication
- C. There must be a zero vector
- D. Every vector must have an additive inverse
- E. Addition must be associative
- F. Addition must be commutative
- G. Scalar multiplication by 1 is the identity operation
- H. The distributive property
- I. Scalar multiplication must be associative
- J. None of the above, it is a vector space
Do the vectors in the set $S$ span the vector space $V$?

- A. Yes
- B. No

If not, why not? (Check all that apply)

- A. W does not contain a zero vector
- B. Not applicable: W is a subspace
- C. W is not closed under addition
- D. W is not closed under multiplication by a nonzero scalar

---

2. (1 pt) $V=\mathbb{R}^2$

$W=\{(x, y)|x^2 + y^2 = 1\}$

Is the given subset $W$ of the vector space $V$ a subspace of $V$?

- A. Yes
- B. No

If not, why not? (Check all that apply)

- A. W does not contain a zero vector
- B. Not applicable: W is a subspace
- C. W is not closed under addition
- D. W is not closed under multiplication by a nonzero scalar

---

3. (1 pt) $V=\mathbb{R}^3$

$S=\{(1, 0, 0), (0, 1, 0), (-5, 1, 2)\}$

Do the vectors in the set $S$ span the vector space $V$?

- A. Yes

---

4. (1 pt) $V=\mathbb{R}^3$

$S=\{(6, 0, 5), (2, 0, -2), (-8, 0, 2), (-2, 0, -3)\}$

Do the vectors in the set $S$ span the vector space $V$?

- A. Yes
- B. No

---

5. (1 pt) Finding Kernels

Find the kernel of the following transformation. Use $a$, $b$, $c$, and $d$ in your answer.

$T: \mathbb{P}_2 \rightarrow \mathbb{P}_2$,

$T(at^2 + bt + c) = 2at + b$

$\quad \boxed{f^2 + f +}$

---

6. (1 pt) Finding Kernels

Find the kernel of the following transformation. Use $a$, $b$, $c$, and $d$ in your answer.

$T: \mathbb{P}_2 \rightarrow \mathbb{P}_2$,

$T(at^2 + bt + c) = 0$

$\quad \boxed{f^3 + f^2 + f +}$

---

7. (1 pt) Which of the following sets are subspaces of $\mathbb{R}^3$?

- A. $\{(2x + 5y, -3x - 4y, -9x + 4y) \mid x, y \text{ arbitrary numbers}\}$
- B. $\{(x, 0, 0) \mid x \text{ arbitrary number}\}$
- C. $\{(x, y, z) \mid x, y, z > 0\}$
- D. $\{(x, y, z) \mid 8x + 7y + 6z = 0\}$
- E. $\{(x, y, z) \mid x + y + z = -2\}$
- F. $\{(8, y, z) \mid y, z \text{ arbitrary numbers}\}$
8. (1 pt) Let \( A = \begin{bmatrix} 0 & -2 \\ -4 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \) and \( C = \begin{bmatrix} 4 \\ 3 \end{bmatrix}. \)

8.1. Determine whether or not the three vectors listed above are linearly independent or linearly dependent.

If they are linearly dependent, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) otherwise, if the vectors are linearly independent, enter 0’s for the coefficients, since that relationship always holds.

\[ A + B + C = 0. \]

You can use this \textbf{row reduction tool} to help with the calculations.

9. (1 pt) Let \( A = \begin{bmatrix} 5 & -2 & 3 \\ 5 & -3 & 6 \end{bmatrix}, B = \begin{bmatrix} -10 \\ 5 \end{bmatrix}) \)

9.1. Determine whether or not the three vectors listed above are linearly independent or linearly dependent.

The vectors were written horizontally this time, as they often are in books, but that is just to save space. The problem is the same as if the vectors were written vertically.

If they are linearly dependent, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) otherwise, if the vectors are linearly independent, enter 0’s for the coefficients, since that relationship always holds.

\[ A + B + C = 0. \]

You can use this \textbf{row reduction tool} to help with the calculations.

10. (1 pt) Let \( A = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 8 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 3 \\ 7 \\ 3 \end{bmatrix}, C = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}, \) and \( D = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \\ 2 \end{bmatrix}. \)

10.1. Determine whether or not the four vectors listed above are linearly independent or linearly dependent.

If they are linearly dependent, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) Otherwise, if the vectors are linearly independent, enter 0’s for the coefficients, since that relationship always holds.

\[ A + B + C + D = 0. \]

You can use this \textbf{row reduction tool} to help with the calculations.

11. (1 pt) Is the following subset \( S \) of \( C(\mathbb{R}) \) (continuous function on the real numbers) linearly independent?\( S=\{e^t, te^t, t^3 e^t\} \)

\[ \begin{align*}
\text{A. Yes} \\
\text{B. No}
\end{align*} \]

12. (1 pt) Consider the set \( \{(1,1)\} \) in \( \mathbb{R}^2 \).

Why isn’t the set a basis for \( \mathbb{R}^2 \)? (check all that apply)

\[ \begin{align*}
\text{A. The vectors are linearly dependent} \\
\text{B. The span of the vectors is not} \mathbb{R}^2 \\
\text{C. None of the above, it is a basis for} \mathbb{R}^2
\end{align*} \]

13. (1 pt) Consider the set \( \{(-7,5),(5,-7)\} \) in \( \mathbb{R}^2 \).

Why isn’t the set a basis for \( \mathbb{R}^2 \)? (check all that apply)

\[ \begin{align*}
\text{A. The vectors are linearly dependent} \\
\text{B. The span of the vectors is not} \mathbb{R}^2 \\
\text{C. None of the above, it is a basis for} \mathbb{R}^2
\end{align*} \]

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Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 2 instead of 8, $\sin(3 \times \pi/2)$ instead of -1, $e \wedge (\ln(2))$ instead of 2, $(2 + \tan(3)) \times (4 - \sin(5)) \wedge 6 - 7/8$ instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

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1. (1 pt) Let $v_1 = \begin{bmatrix} -1 \\ 5 \\ -4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -17 \\ 15 \end{bmatrix}$, and $y = \begin{bmatrix} -4 \\ 24 \\ h \end{bmatrix}$.

For what value of $h$ is $y$ in the plane spanned by $v_1$ and $v_2$? $h =$ __________

---

2. (1 pt) Which of the following sets of vectors are linearly independent?

- A. $\begin{bmatrix} 4 \\ -5 \\ -7 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 0 \\ -3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -8 \\ 8 \\ 3 \end{bmatrix}$
- B. $\begin{bmatrix} 4 \\ 7 \\ 6 \\ -2 \\ 8 \\ 3 \end{bmatrix}$
- C. $\begin{bmatrix} 7 \\ 2 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
- D. $\begin{bmatrix} 18 \\ -16 \\ -9 \\ 8 \end{bmatrix}$
- E. $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 7 \end{bmatrix}$
- F. $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

---

3. (1 pt) Express the vector $v = \begin{bmatrix} 0 \\ -22 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $y = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

$v = \_ x + \_ y$

---

4. (1 pt) Determine whether the given set $S$ is a subspace of the vector space $V$.

- A. $V = M_n(\mathbb{R})$, and $S$ is the subset of all $n \times n$ matrices with $\det(A) = 0$.
- B. $V = C^2(I)$, and $S$ is the subset of $V$ consisting of those functions satisfying the differential equation $y'' - 4y' + 3y = 0$.
- C. $V = P_3$, and $S$ is the subset of $P_3$ consisting of all polynomials of the form $p(x) = x^2 + c$.
- D. $V = \mathbb{R}^2$, and $S$ consists of all vectors $(x_1, x_2)$ satisfying $x_1^2 - x_2^2 = 0$.
- E. $V = P_4$, and $S$ is the subset of $P_4$ consisting of all polynomials of the form $p(x) = ax^3 + bx$.
- F. $V$ is the vector space of all real-valued functions defined on the interval $(-\infty, \infty)$, and $S$ is the subset of $V$ consisting of those functions satisfying $f(0) = 0$.
- G. $V = \mathbb{R}^n$, and $S$ is the set of solutions to the homogeneous linear system $Ax = 0$ where $A$ is a fixed $m \times n$ matrix.

---

5. (1 pt) Which of the following subsets of $\mathbb{R}^{3 \times 3}$ are subspaces of $\mathbb{R}^{3 \times 3}$?

- A. The diagonal $3 \times 3$ matrices
- B. The non-invertible $3 \times 3$ matrices
- C. The invertible $3 \times 3$ matrices
- D. The $3 \times 3$ matrices with all zeros in the third row
- E. The symmetric $3 \times 3$ matrices
- F. The $3 \times 3$ matrices whose entries are all greater than or equal to 0

---

6. (1 pt) Determine if each of the following sets is a subspace of $\mathbb{P}_n$, for an appropriate value of $n$. Type “yes” or “no” for each answer.

Let $W_1$ be the set of all polynomials of the form $p(t) = at^2$, where $a$ is in $\mathbb{R}$. ______

Let $W_2$ be the set of all polynomials of the form $p(t) = t^2 + a$, where $a$ is in $\mathbb{R}$. ______

Let $W_3$ be the set of all polynomials of the form $p(t) = at^2 + at$, where $a$ is in $\mathbb{R}$. ______

---

7. (1 pt) Determine whether the given set $S$ is a subspace of the vector space $V$.

- A. $V = M_n(\mathbb{R})$, and $S$ is the subset of all nonsingular matrices.
• B. $V = P_3$, and $S$ is the subset of $P_3$ consisting of all polynomials of the form $p(x) = ax^3 + bx$.

• C. $V = C^3(I)$, and $S$ is the subset of $V$ consisting of those functions satisfying the differential equation $y''' + 6y = x^2$.

• D. $V = P_5$, and $S$ is the subset of $P_5$ consisting of those polynomials satisfying $p(1) > p(0)$.

• E. $V = M_n(\mathbb{R})$, and $S$ is the subset of all symmetric matrices.

• F. $V = C^5(I)$, and $S$ is the subset of $V$ consisting of those functions satisfying the differential equation $y^{(5)} = 0$.

• G. $V$ is the vector space of all real-valued functions defined on the interval $(-\infty, \infty)$, and $S$ is the subset of $V$ consisting of those functions satisfying $f(0) = 0$.

8. (1 pt) Which of the following subsets of $P_2$ are subspaces of $P_2$?

• A. $\{ p(t) \mid p'(t) \text{ is constant} \}$

• B. $\{ p(t) \mid p'(t) + 8p(t) + 3 = 0 \}$

• C. $\{ p(t) \mid \int_0^3 p(t)dt = 0 \}$

• D. $\{ p(t) \mid p(4) = 0 \}$

• E. $\{ p(t) \mid p(8) = 1 \}$

• F. $\{ p(t) \mid p(-t) = p(t) \text{ for all } t \}$

9. (1 pt) Which of the following sets are subspaces of $\mathbb{R}^3$?

• A. $\{ (-7, y, z) \mid y, z \text{ arbitrary numbers} \}$

• B. $\{ (x, y, z) \mid 7x - 9y = 0, -6x - 3z = 0 \}$

• C. $\{ (x,y,z) \mid x,y,z > 0 \}$

• D. $\{ (-3x, -5x, 4x) \mid x \text{ arbitrary number} \}$

• E. $\{ (x,y,z) \mid x + y + z = 3 \}$

• F. $\{ (x,y,z) \mid -7x + 9y + 6z = 0 \}$
Gauchospace link

The course contains the syllabus, grading policy and other information.

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Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as 2 \^ 3 instead of 8, sin(3 * pi/2) instead of -1, e \& (ln(2)) instead of 2, (2 + tan(3)) \* (4 – sin(5)) \& 6 – 7/8 instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

1. (1 pt) Let \( W_1 \) be the set: \[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix},
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}.
\]
Determine if \( W_1 \) is a basis for \( \mathbb{R}^3 \) and check the correct answer(s) below.

- A. \( W_1 \) is not a basis because it is linearly dependent.
- B. \( W_1 \) is not a basis because it does not span \( \mathbb{R}^3 \).
- C. \( W_1 \) is a basis.

Let \( W_2 \) be the set: \[
\begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix},
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix},
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}.
\]
Determine if \( W_2 \) is a basis for \( \mathbb{R}^3 \) and check the correct answer(s) below.

- A. \( W_2 \) is not a basis because it is linearly dependent.
- B. \( W_2 \) is a basis.
- C. \( W_2 \) is not a basis because it does not span \( \mathbb{R}^3 \).

2. (1 pt) Let \( A = \) \[
\begin{bmatrix}
3 & -5 & 11 \\
5 & 2 & 12 \\
2 & -5 & 9 \\
-4 & 8 & 0 \\
3 & 7 & -1
\end{bmatrix}
\]
Give a basis for the column space of \( A \).

\( u = \) \[
\begin{bmatrix}
-1 \\
-1 \\
-1
\end{bmatrix},
\( v = \) \[
\begin{bmatrix}
-1 \\
-1 \\
-1
\end{bmatrix}.
\]

3. (1 pt) Determine whether each set \( \{ p_1, p_2 \} \) is a linearly independent set in \( \mathbb{P}_3 \). Type "yes" or "no" for each answer.

The polynomials \( p_1(t) = 1 + t^2 \) and \( p_2(t) = 1 - t^2 \). ____

The polynomials \( p_1(t) = 2t + t^2 \) and \( p_2(t) = 1 + t \). ____

The polynomials \( p_1(t) = 2t - 4t^2 \) and \( p_2(t) = 6t^2 - 3t \). ____

4. (1 pt) Check the true statements below:

- A. In some cases, the linear dependence relations among the columns of a matrix can be affected by certain elementary row operations on the matrix.
- B. A single vector by itself is linearly dependent.
- C. A basis is a spanning set that is as large as possible.
- D. The columns of an invertible \( n \times n \) matrix form a basis for \( \mathbb{R}^n \).
- E. If \( H = Span\{b_1,...,b_p\} \), then \( \{b_1,...,b_p\} \) is a basis for \( H \).

5. (1 pt) Let \( W_1 \) be the set: \[
\begin{bmatrix}
1 \\
-3 \\
0
\end{bmatrix},
\begin{bmatrix}
-2 \\
9 \\
0
\end{bmatrix},
\begin{bmatrix}
0 \\
0 \\
-3
\end{bmatrix},
\begin{bmatrix}
0 \\
5
\end{bmatrix}.
\]
Determine if \( W_1 \) is a basis for \( \mathbb{R}^3 \) and check the correct answer(s) below.

- A. \( W_1 \) is not a basis because it is linearly dependent.
- B. \( W_1 \) is not a basis because it does not span \( \mathbb{R}^3 \).
- C. \( W_1 \) is a basis.

Let \( W_2 \) be the set: \[
\begin{bmatrix}
-2 \\
3 \\
0
\end{bmatrix},
\begin{bmatrix}
6 \\
-1 \\
5
\end{bmatrix}.
\]
Determine if \( W_2 \) is a basis for \( \mathbb{R}^3 \) and check the correct answer(s) below.

- A. \( W_2 \) is a basis.
- B. \( W_2 \) is not a basis because it is linearly dependent.
- C. \( W_2 \) is not a basis because it does not span \( \mathbb{R}^3 \).

6. (1 pt) Show that the vectors \( \langle 1,2,1 \rangle, \langle 1,3,1 \rangle, \langle 1,4,1 \rangle \) do not span \( \mathbb{R}^3 \) by giving a vector not in their span: ______
7. (1 pt)
Let \( S = \{r, u, d, x\} \) be a set of vectors.

If \( x = r + 3u + d \), determine whether or not \( S \) is linearly independent.

\[ ? \]

Determine whether or not the four vectors listed above are linearly independent or linearly dependent.

If \( S \) is dependent, enter a non-trivial linear relation below. Otherwise, enter 0’s for the coefficients.

\[ r \text{ } + \text{ } u \text{ } + \text{ } d \text{ } + \text{ } x = 0. \]

8. (1 pt)
Suppose \( S = \{r, u, d\} \) is a set of linearly independent vectors.

If \( x = 3r + 4u + d \), determine whether \( T = \{r, u, x\} \) is a linearly independent set.

\[ ? \]

Is \( T \) linearly independent or dependent?

If \( T \) is dependent, enter a non-trivial linear relation below. Otherwise, enter 0’s for the coefficients.

\[ r \text{ } + \text{ } u \text{ } + \text{ } x = 0. \]
WeBWorK assignment number setWeek7-1 is due: 02/26/2014 at 04:00am PST.

The Gauchospace link for the course contains the syllabus, grading policy and other information.

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1. (1 pt) Find a basis of the subspace of \( \mathbb{R}^4 \) spanned by the following vectors:

\[
\begin{bmatrix}
-4 \\ -3 \\ -2 \\ -4 \\
14
\end{bmatrix},
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\
0
\end{bmatrix},
\begin{bmatrix}
-1 \\ -4 \\ 6 \\ -1 \\
10
\end{bmatrix},
\begin{bmatrix}
5 \\ 7 \\ -3 \\ 5 \\
-24
\end{bmatrix},
\begin{bmatrix}
-5 \\ -7 \\ 3 \\ -5 \\
24
\end{bmatrix}.
\]

2. (1 pt) Find a basis of the subspace of \( \mathbb{R}^4 \) defined by the equation \( 3x_1 - 9x_2 + 2x_3 + 9x_4 = 0 \).

3. (1 pt) Find a basis for the space of \( 2 \times 2 \) lower triangular matrices: \( \{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \} \)

4. (1 pt) Find a linearly independent set of vectors that spans the same subspace of \( \mathbb{R}^3 \) as that spanned by the vectors

\[
\begin{bmatrix}
-3 \\ -2 \\ -3
\end{bmatrix},
\begin{bmatrix}
2 \\ 0 \\ 6
\end{bmatrix},
\begin{bmatrix}
2 \\ 2 \\ 0
\end{bmatrix}.
\]

Linearly independent set: \( \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \).

5. (1 pt) Changing Coordinates

Let \( S \) be the standard basis and \( B = b_1, b_2 = \begin{bmatrix} -4 \\ 3 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \) be a new basis for \( \mathbb{R}^2 \).

Convert the following vectors to the basis \( B \)

\[
\begin{bmatrix} 3 \\ 8 \\ 2 \\ 0 \end{bmatrix} \] changes to \( \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \)

Calculate the coordinate change matrixes \( M_B \) to go from \( B \) to \( S \) and \( M_B^{-1} \)

\[
M_B = \begin{bmatrix} \_ \_ \_ \\ \_ \_ \_ \_ \\ \_ \_ \_ \_ \end{bmatrix},
M_B^{-1} = \begin{bmatrix} \_ \_ \\ \_ \_ \_ \\ \_ \_ \_ \_ \end{bmatrix}
\]
8. (1 pt) Changing Coordinates
Let S be the standard basis and
\[ B = b_1, b_2 = \begin{bmatrix} 3 \\ -2 \\ -4 \\ 3 \end{bmatrix} \]
be a new basis for \( \mathbb{R}^2 \)

The following vectors are in basis B. Convert them to their standard basis representations.
\[ \begin{bmatrix} 3 \\ -1 \\ 2 \\ 1 \end{bmatrix} \text{ changes to } \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \]
\[ \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} \text{ changes to } \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \]

9. (1 pt) Changing Coordinates
Let S be the standard basis and
\[ B = b_1, b_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 2 \end{bmatrix} \]
be a new basis for \( \mathbb{R}^2 \)

Calculate the coordinate change matrixes \( M_B \) to go from B to S and \( M_B^{-1} \)
\[ M_B = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \]
\[ M_B^{-1} = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \]

10. (1 pt) Changing Coordinates
Let S be the standard basis and
\[ B = b_1, b_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 2 \end{bmatrix} \]
be a new basis for \( \mathbb{R}^2 \)

Convert the following vectors to the basis B.
\[ \begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \end{bmatrix} \text{ changes to } \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \]
\[ \begin{bmatrix} -1 \\ 1 \\ 4 \\ 5 \end{bmatrix} \text{ changes to } \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \]

11. (1 pt) Changing Coordinates
Let S be the standard basis and
\[ B = b_1, b_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 2 \end{bmatrix} \]
be a new basis for \( \mathbb{R}^2 \)

The following vectors are in basis B. Convert them to their standard basis representations.
\[ \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} \text{ changes to } \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \]
\[ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \text{ changes to } \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \]
The Gauchospace link

for the course contains the syllabus, grading policy and other information.

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1. (1 pt) The vectors

\[
v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 7 \\ 5 \end{bmatrix}, \quad \text{and} \quad v_3 = \begin{bmatrix} 4 \\ 24 \\ k \end{bmatrix}
\]

form a basis for \( \mathbb{R}^3 \) if and only if \( k \neq \_ \).

2. (1 pt) Consider the basis \( B \) of \( \mathbb{R}^2 \) consisting of vectors

\[
\begin{bmatrix} -1 \\ 4 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -6 \\ -7 \end{bmatrix}.
\]

Find \( x \in \mathbb{R}^2 \) whose coordinate vector relative to the basis \( B \) is

\[
[x]_B = \begin{bmatrix} -6 \\ -3 \end{bmatrix}.
\]

3. (1 pt) The set \( \{ \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -12 \\ 14 \\ 0 \end{bmatrix} \} \) is a basis for \( \mathbb{R}^2 \).

Find the coordinates of the vector \( x = \begin{bmatrix} -44 \\ 46 \end{bmatrix} \) relative to the basis \( B \):

\[
[x]_B = \begin{bmatrix} \_ \\ \_ \end{bmatrix}.
\]

4. (1 pt) Find the coordinate vector of \( x = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \) with respect to the basis \( B = \{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \} \) or \( \mathbb{R}^3 \).

\[
[x]_B = \begin{bmatrix} \_ \\ \_ \end{bmatrix}.
\]

5. (1 pt) Let \( A = \begin{bmatrix} -4 & -8 & 4 \\ 1 & 2 & -1 \end{bmatrix} \).

Find bases of the kernel and image of \( A \) (or the linear transformation \( T(x) = Ax \)).

Kernel:

\[
\begin{bmatrix} \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \end{bmatrix}.
\]

Image:

\[
\begin{bmatrix} \_ \end{bmatrix}.
\]

6. (1 pt) Let \( A = \begin{bmatrix} -3 & 2 \\ -9 & 6 \\ 9 & -6 \end{bmatrix} \).

Find bases of the kernel and image of \( A \) (or the linear transformation \( T(x) = Ax \)).

Kernel:

\[
\begin{bmatrix} \_ \end{bmatrix}.
\]

Image:

\[
\begin{bmatrix} \_ \end{bmatrix}.
\]

7. (1 pt) Let \( A = \begin{bmatrix} -3 & -2 & 2 \\ 3 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \).

Find a basis of nullspace(\( A \)).

\[
\begin{bmatrix} \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \end{bmatrix}.
\]

8. (1 pt) Let \( A = \begin{bmatrix} -9 & -4 & -2 \\ -3 & 1 & 4 \\ -3 & 1 & 4 \end{bmatrix} \).

Find a basis of the image of \( A \) (or the linear transformation \( T(x) = Ax \)).

\[
\begin{bmatrix} \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \end{bmatrix}.
\]
WeBWorK assignment number set Week 8-1 is due: 03/05/2014 at 03:00am PST.

The Gauchospace link for the course contains the syllabus, grading policy and other information.

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You can use the Feedback button on each problem page to send e-mail to the professors.

1. (1 pt) Find a basis of the subspace of $\mathbb{R}^4$ spanned by the following vectors:

\[
\begin{pmatrix}
2 \\
-1 \\
4 \\
a \\
\end{pmatrix},
\begin{pmatrix}
1 \\
1 \\
2 \\
b \\
\end{pmatrix},
\begin{pmatrix}
3 \\
-3 \\
6 \\
c \\
\end{pmatrix},
\begin{pmatrix}
8 \\
-1 \\
16 \\
d \\
\end{pmatrix}
\]

2. (1 pt) Find a basis of the column space of the matrix

\[
A = \begin{bmatrix}
-2 & -4 & 0 & 4 \\
3 & 3 & 4 & 1 \\
8 & 10 & 8 & 6 \\
\end{bmatrix}
\]

3. (1 pt) Find the eigenvalues of the matrix

\[
A = \begin{bmatrix}
6 & -2 \\
4 & 0 \\
\end{bmatrix}
\]

The smaller eigenvalue is $\lambda_1 = \ldots$. 
The bigger eigenvalue is $\lambda_2 = \ldots$.

4. (1 pt) The matrix $C = \begin{bmatrix}
3 & 0 & 0 \\
4 & 1 & 0 \\
-9 & -3 & -2 \\
\end{bmatrix}$ has three distinct eigenvalues, $\lambda_1 < \lambda_2 < \lambda_3$, where $\lambda_1 = \ldots$, $\lambda_2 = \ldots$, and $\lambda_3 = \ldots$.

5. (1 pt) Find the eigenvalues of the matrix

\[
A = \begin{bmatrix}
27 & -33 \\
22 & -28 \\
\end{bmatrix}
\]

smaller eigenvalue = \ldots
associated eigenvector = ( , ).
larger eigenvalue = \ldots
associated eigenvector = ( , ).

6. (1 pt) The matrix $A = \begin{bmatrix}
-3 & k \\
-7 & -7 \\
\end{bmatrix}$ has two distinct real eigenvalues if and only if $k < \ldots$.

7. (1 pt) The matrix $A = \begin{bmatrix}
0 & 0 & -1 & 1 \\
0 & -2 & -2 & 2 \\
2 & 0 & -1 & -1 \\
2 & 0 & -1 & -1 \\
\end{bmatrix}$ has two distinct eigenvalues $\lambda_1 < \lambda_2$. Find the eigenvalues and a basis of each eigenspace.

$\lambda_1 = \ldots$
Basis: [ , ]

$\lambda_2 = \ldots$
Basis: [ , ]

8. (1 pt) Give an example of a 2x2 matrix without any real eigenvalues:

\[
\begin{bmatrix}
\ldots & \ldots \\
\ldots & \ldots \\
\end{bmatrix}
\]

9. (1 pt) Let $M = \begin{bmatrix}
8 & -2 \\
4 & 2 \\
\end{bmatrix}$.
Find formulas for the entries of $M^n$, where $n$ is a positive integer.

$M^n = \begin{bmatrix}
\ldots & \ldots \\
\ldots & \ldots \\
\end{bmatrix}$

10. (1 pt) Let $M = \begin{bmatrix}
11 & 1 \\
-49 & -3 \\
\end{bmatrix}$.
Find formulas for the entries of $M^n$, where $n$ is a positive integer.

$M^n = \begin{bmatrix}
\ldots & \ldots \\
\ldots & \ldots \\
\end{bmatrix}$
11. (1 pt) Let: \( A = \begin{bmatrix} -8 & 0.25 \\ 25 & -8 \end{bmatrix} \)

Find an invertible \( S \) and a diagonal \( D \) such that \( S^{-1}AS = D \).

\[ S = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \quad D = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \]

12. (1 pt) **More Eigenstuff**

Calculate the eigenvalues and a basis for its eigenspace, ie. its eigenvector(s).

Fill any unused box with a 0.

\[ \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} \]

Least Eigenvalue: \( \lambda_1 = \)

Eigenspace: \( E_{\lambda_1} = \text{span} \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix} \)

Next Eigenvalue: \( \lambda_2 = \)

Eigenspace: \( E_{\lambda_2} = \text{span} \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix} \)

Greatest Eigenvalue: \( \lambda_3 = \)

Eigenspace: \( E_{\lambda_3} = \text{span} \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix} \)

¡!
1. (1 pt) Given that \( \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) and \( \mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \) are eigenvectors of the matrix \( A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & 3 \\ -5 & 5 & 1 \end{bmatrix} \), determine the corresponding eigenvalues. 
\[ \lambda_1 = \ldots, \quad \lambda_2 = \ldots. \]

2. (1 pt) If \( \mathbf{v}_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \) and \( \mathbf{v}_2 = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \) are eigenvectors of a matrix \( A \) corresponding to the eigenvalues \( \lambda_1 = -4 \) and \( \lambda_2 = 5 \), respectively, then \( A(\mathbf{v}_1 + \mathbf{v}_2) = \begin{bmatrix} \ldots & \ldots \end{bmatrix} \) and \( A(3\mathbf{v}_1) = \begin{bmatrix} \ldots & \ldots \end{bmatrix} \).

3. (1 pt) Suppose \( A \) is an invertible \( n \times n \) matrix and \( v \) is an eigenvector of \( A \) with associated eigenvalue 4. Convince yourself that \( v \) is an eigenvalue of the following matrices, and find the associated eigenvalues:
1. \( A^2 \), eigenvalue = \ldots.
2. \( A^{-1} \), eigenvalue = \ldots.
3. \( A + 4I_n \), eigenvalue = \ldots.
4. \( 7A \), eigenvalue = \ldots.

4. (1 pt) \( A \) is an \( n \times n \) matrix.

Check the true statements below:

- A. To find the eigenvalues of \( A \), reduce \( A \) to echelon form.
- B. A matrix \( A \) is not invertible if and only if 0 is an eigenvalue of \( A \).
- C. A number \( c \) is an eigenvalue of \( A \) if and only if the equation \( (A - cI)x = 0 \) has a nontrivial solution \( x \).
- D. If \( Ax = \lambda x \) for some vector \( x \), then \( \lambda \) is an eigenvalue of \( A \).
- E. Finding an eigenvector of \( A \) might be difficult, but checking whether a given vector is in fact an eigenvector is easy.

5. (1 pt) The matrix \( A = \begin{bmatrix} -5 & 0 & 8 \\ 4 & -1 & -8 \\ -2 & 0 & 3 \end{bmatrix} \) has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace. 
\[ \text{eigenvalue} = \ldots, \quad \text{multiplicity} = \ldots, \quad \text{dimension of the eigenspace} = \ldots. \]

6. (1 pt) A matrix \( A \) such that
\[ \begin{bmatrix} 2 \\ 1 \\ -3 \\ -2 \end{bmatrix} \] and \[ \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \] are eigenvectors of \( A \), with eigenvalues 8 and -7 respectively. 
\[ A = \begin{bmatrix} \ldots & \ldots \end{bmatrix}. \]

7. (1 pt) Find a basis of the eigenspace associated with the eigenvalue -2 of the matrix \( A = \begin{bmatrix} -2 & 0 & 2 \\ 2 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix} \). 
\[ \begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix} \]

8. (1 pt) The matrix \( B = \begin{bmatrix} 9 & 6 & 3 \\ 0 & -6 & -9 \\ 0 & 0 & 3 \end{bmatrix} \) has three distinct eigenvalues, \( \lambda_1 < \lambda_2 < \lambda_3 \), where \( \lambda_1 = \ldots, \lambda_2 = \ldots, \text{ and } \lambda_3 = \ldots. \)

9. (1 pt) The matrix \( C = \begin{bmatrix} -6 & 4 & 0 \\ -2 & 0 & 0 \\ -2 & 2 & -2 \end{bmatrix} \) has two distinct eigenvalues, \( \lambda_1 < \lambda_2 \): 
\[ \lambda_1 = \ldots \text{ has multiplicity } \ldots \text{ and } \lambda_2 = \ldots \text{ has multiplicity } \ldots. \]
1. (1 pt) Let \(x = \begin{bmatrix} -2 \\ -3 \\ 5 \end{bmatrix}\) and \(y = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}\). Find the dot product of \(x\) and \(y\).

\[x \cdot y = \ldots\]

2. (1 pt) Let \(x = \begin{bmatrix} 1 \\ -2 \\ -5 \\ -5 \end{bmatrix}\). Find the norm of \(x\) and the unit vector in the direction of \(x\).

\[|x| = \ldots, \quad u = \begin{bmatrix} \ldots \\ \ldots \\ \ldots \\ \ldots \end{bmatrix}\]

3. (1 pt) Find a vector \(v\) perpendicular to the vector \(u = \begin{bmatrix} 8 \\ -8 \end{bmatrix}\).

\[v = \begin{bmatrix} \ldots \\ \ldots \end{bmatrix}\]

4. (1 pt) Let \(x = \begin{bmatrix} -4 \\ 9 \end{bmatrix}\) and \(y = \begin{bmatrix} -6 \\ -2 \end{bmatrix}\). Find the dot product of \(x\) and \(y\).

\[x \cdot y = \ldots\]

5. (1 pt) Find the length of the vector \(x = \begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}\).

\[|x| = \ldots\]

6. (1 pt) Find a vector \(x\) perpendicular to the vectors \(v = \begin{bmatrix} -3 \\ -8 \\ 2 \end{bmatrix}\) and \(u = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}\).

\[x = \begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix}\]

7. (1 pt) Find two linearly independent vectors perpendicular to the vector \(v = \begin{bmatrix} -2 \\ 5 \\ -1 \end{bmatrix}\).

\[\begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix}, \quad \begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix}\]

8. (1 pt)

Calculate the following dot product: \([9, 7] \cdot [-7, 9]\)

Are these vectors orthogonal?

- A. Yes
- B. No

9. (1 pt)

Calculate the following dot product: \([7, 7, 7, 1] \cdot [-2, 3, -3, 14]\)

Are these vectors orthogonal?

- A. Yes
- B. No

10. (1 pt) Find the least-squares solution \(x^*\) of the system

\[
\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} -8 \\ -1 \\ 4 \end{bmatrix}.
\]

\[x^* = \begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix}\]

11. (1 pt) Find the least-squares solution \(x^*\) of the system

\[
\begin{bmatrix} 6 & -3 & -2 \\ -3 & -10 & -8 \end{bmatrix} x = \begin{bmatrix} 25 \\ \ldots \\ \ldots \end{bmatrix}.
\]

\[x^* = \begin{bmatrix} \ldots \\ \ldots \\ \ldots \end{bmatrix}\]
12. (1 pt) Find the least-squares solution $x^*$ of the system
\[
\begin{bmatrix}
2 & -2 \\
-2 & 2 \\
4 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
x
\end{bmatrix}
= 
\begin{bmatrix}
2 \\
-10 \\
-23
\end{bmatrix}
.
\]
$x^* =$

13. (1 pt) Find the least-squares solution $x^*$ of the system
\[
\begin{bmatrix}
1 & -1 & -1 \\
1 & 1 & -1 \\
1 & -1 & 1 \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
x \\
x
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
1 \\
-7 \\
-11
\end{bmatrix}
.
\]
x^* =

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The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA’s or your professor for help. Don’t spend a lot of time guessing – it’s not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as \(2 \cdot 3\) instead of 8, \(\sin(3 \cdot \pi/2)\) instead of -1, \(e \cdot (\ln(2))\) instead of 2, \((2 + \tan(3)) \cdot (4 - \sin(5)) \cdot 6 - 7/8\) instead of 27620.3413, etc. Here’s the list of the functions which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

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1. (1 pt) Suppose \(v_1, v_2, v_3\) is an orthogonal set of vectors in \(\mathbb{R}^3\). Let \(w\) be a vector in \(\text{Span}(v_1, v_2, v_3)\) such that
\[
v_1 \cdot v_1 = 14, v_2 \cdot v_2 = 27, v_3 \cdot v_3 = 4, \\
v_1 \cdot v_2 = -70, v_2 \cdot v_3 = -135, v_1 \cdot v_3 = -8,
\]
then \(w = ______v_1 + ______v_2 + ______v_3\).

2. (1 pt) Given \(v = \begin{bmatrix} -1 \\ 8 \end{bmatrix}\), find the coordinates for \(v\) in the subspace \(W\) spanned by \(u_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\) and \(u_2 = \begin{bmatrix} 4 \\ -4 \end{bmatrix}\). Note that \(u_1\) and \(u_2\) are orthogonal.
\[
v = ______u_1 + ______u_2
\]

3. (1 pt) Given \(v = \begin{bmatrix} 1 \\ -5 \\ -10 \end{bmatrix}\), find the coordinates for \(v\) in the subspace \(W\) spanned by \(u_1 = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}\), \(u_2 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}\) and 
\[
u_3 = \begin{bmatrix} 9 \\ -10 \\ 3 \end{bmatrix}. \text{Note that } u_1, u_2 \text{ and } u_3 \text{ are orthogonal.}
\]
\[
v = ______u_1 + ______u_2 + ______u_3
\]

4. (1 pt) Given \(v = \begin{bmatrix} -10 \\ -10 \\ 3 \\ 4 \\ -6 \end{bmatrix}\), find the coordinates for \(v\) in the subspace \(W\) spanned by \(u_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}\), \(u_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}\), 
\[
u_3 = \begin{bmatrix} -7 \\ 3 \\ 5 \\ 16 \\ 5 \end{bmatrix}, \text{and } u_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} \text{ and } u_5 = \begin{bmatrix} -4 \\ -2 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \text{Note that } u_1, u_2,
\]
\[
u_3, u_4 \text{ and } u_5 \text{ are orthogonal.}
\]
\[
v = ______u_1 + ______u_2 + ______u_3 + ______u_4 + ______u_5
\]

5. (1 pt) Find the eigenvalues of the following matrix.
\[
A = \begin{bmatrix}
-19 & 12 & 0 & 0 \\
-16 & 9 & 0 & 0 \\
0 & 0 & -8 & -12 \\
0 & 0 & 16 & 20
\end{bmatrix}
\]
The eigenvalues are \(\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4\), where \(\lambda_1 = \ldots\), associated eigenvector = (____, ____), \(\lambda_2 = \ldots\), associated eigenvector = (____, ____), \(\lambda_3 = \ldots\), associated eigenvector = (____, ____), \(\lambda_4 = \ldots\), associated eigenvector = (____, ____).

6. (1 pt) Find the eigenvectors of \(A\), given that \(A = \begin{bmatrix} 3 \\ -7 \\ -13 \\ 0 \\ -2 \\ 1 \\ -1 \\
1 \end{bmatrix}\)
and its eigenvalues are \(\lambda_1 = -3\) with multiplicity 2 and \(\lambda_2 = 3\) with multiplicity 1.
\[
\lambda_1 \text{ associated eigenvector = (____, ____),} \\
\lambda_2 \text{ associated eigenvector = (____, ____).}
\]

7. (1 pt) \(A\) is an \(m \times n\) matrix.

Check the true statements below:

- A. The null space of \(A\) is the solution set of the equation \(Ax = 0\).
- B. The column space of \(A\) is the range of the mapping \(x \rightarrow Ax\).
- C. The kernel of a linear transformation is a vector space.
- D. \(\text{Col}A\) is the set of all vectors that can be written as \(Ax\) for some \(x\).
- E. The null space of an \(m \times n\) matrix is in \(\mathbb{R}^m\).
F. If the equation $Ax = b$ is consistent, then $ColA$ is $\mathbb{R}^m$.

8. (1 pt) Let $A = \begin{bmatrix} -4 & -4 & 0 \\ 2 & 2 & 0 \\ -2 & -2 & 0 \\ -4 & -4 & 0 \end{bmatrix}$.

Find dimensions of the kernel and image of $A$ (or the linear transformation $T(x) = Ax$).

$\text{dim(Ker}(A)) = \hspace{1cm}$.

$\text{dim(Im}(A)) = \hspace{1cm}$.