Homework 4 Hints

Problems from Crossley’s Book

5.1 The simplest function to use would be an affine function (i.e. a function of the form \( f(x) = mx + b \), where \( m \) and \( b \) are constants). To find a formula for \( f(x) \), first construct the appropriate straight line in \( \mathbb{R}^2 \), then derive the equation \( y = mx + b \) for this line. After finding \( f(x) \), the inverse function should be extremely easy to calculate.

5.5 For stereographic projection of \( S^1 \): Let \((x, y)\) be a point in \( S^1 \setminus \{(0, 1)\} \). Derive an equation for the line passing through \((0, 1)\) and \((x, y)\); you might want to use variables like \( x_1, x_2 \) for this equation. Then calculate the appropriate intersection point of this line with the projection line \( \{(x_1, -1) : x_1 \in \mathbb{R}\} \). Draw a diagram of \( S^1 \) and the projection line in \( \mathbb{R}^2 \) to understand the geometry of this problem.

For stereographic projection of \( S^2 \): Let \((x, y, z)\) be a point in \( S^2 \setminus \{(0, 0, 1)\} \). Derive parametric equations for the line passing through \((0, 0, 1)\) and \((x, y, z)\); you might want to use variables like \( x_1, x_2, x_3 \) for the parametric equations. Then calculate the appropriate intersection point of this line with the projection plane \( \{(x_1, x_2, -1) : x_1, x_2 \in \mathbb{R}\} \). Draw a diagram of \( S^2 \) and the projection plane in \( \mathbb{R}^3 \) to understand the geometry of this problem.

Problems from Hatcher’s Notes

3. Given an open covering \( \{O_\alpha\} \) of \( X \), use the complement of one of the \( O_\alpha \)'s to construct a finite subcovering.
6. It is straightforward to show that $A \cup B$ is compact.

To show that $A \cap B$ is compact (now assuming that $X$ is Hausdorff), use the fact that $A \cap B$ lies in the compact Hausdorff space $A$ (or $B$), and use the relationships between closed sets and compact sets in a compact Hausdorff space. See propositions on pages 33 and 35 of Hatcher’s notes; these propositions were also discussed in class. Working with subspaces might be tricky, so read the lemma (and its proof) on page 11 of Hatcher’s notes.