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MAKING LARGE LECTURES EFFECTIVE:
AN EFFORT TO INCREASE STUDENT SUCCESS

1. INTRODUCTION

As is the case with many college and university professors, I regularly 'teach' groups of 100 or more students gathered together several times each week in a lecture hall. More precisely, I give lectures on the relevant material, supervise the work of graduate teaching assistants, and assign course grades intended to indicate the level of achievement of those enrolled in the class. These are to be accomplished in a manner satisfactory to the students, at least as measured by their evaluations of the course or reflected by an absence of complaints to the chair or dean. Without any concrete course goals or departmental standard of achievement, I am the sole arbiter of the grading standard applied in the course. The identification of legitimate and concrete measures of effectiveness with which to evaluate the course is left to me. Since the course 'product' is, arguably, the mathematical achievement of the students, I believe that any measure of success must be principally constructed of measures of student performance. This, then, is where I start in trying to figure out how to teach any course. It is this process that I wish to discuss in this report on my continuing effort to increase the effectiveness of my teaching in large classes, (see Millett, 1996, 1997 and 1999).

While the following maxim may be debated by many college and university faculty members, the foundation of my teaching efforts is the statement: "If students haven’t learned, you’ve not taught." Beginning with an assessment of the context, I try to establish statements of the course goals that are understandable and with which my students and I can determine whether or not they have been achieved. Depending upon these goals and the context, instructional strategies are selected. Of course, these are often modified during the instruction to take experience into account. For this purpose, lots of information is collected throughout the course. The success or failure of the implementation is determined by this data as are the final grades. An analysis of this data and the experience with the students provides the basis of modifications of the course goals and strategies in the future.

In this note, I describe the experience with a 'pre-calculus' class during the Winter Quarter period, January through March, 2000, following the approach described in the previous paragraph. Although I had not taught this course for quite a while, I am scheduled to teach it again next year. This course provides the occasion for a review of some strategies applied to large class situations, on one hand, and will allow me to describe how one can try to systematically improve the effectiveness of such courses. First I will describe the context and the goals set for this course. Next I will describe the materials and the instructional strategies

Derek Holton et al. (ed.), The Teaching and Learning of Mathematics at University Level: An ICMI Study. 1—16.
employed in the effort to achieve the goals. Finally, I will discuss the outcome of this effort and what this suggests to me about changes to make when I next teach the course in January 2001.

2. CONTEXT

During the Winter Quarter, 2000, at the University of California, Santa Barbara (UCSB), I was the instructor for Mathematics 15, Precalculus. The official objective of the course is to prepare students for either a social and life science calculus course or a physical science and engineering calculus course. Nevertheless successful completion of the course, with a grade of C or better, is not sufficient to enter the physical science course. Nor is it required for entrance to the social and life science calculus course. For the former, a gateway examination must be passed with a minimum score. Taking the exam is required for the social and life science course but no minimum score is required as there is substantial emphasis on pre-calculus material during the first quarter of that course. In a new policy decision, future enrolment in Precalculus will require a minimum score on the gateway exam. To further complicate a student's enrolment decision, the departmental policy recommends pre-calculus enrolment for many students who have exceeded the standard required for enrolment in the physical science and engineering calculus course. The students enrolling in Precalculus represent a wide range of UCSB degree programs, the largest category being 'undeclared.' Since many UCSB students explore but do not select majors prior to their third year of university, they are commonly thought to enrol in the course in order to provide a calculus option if required for their major or to prepare for other quantitatively oriented courses such as statistics that are required in their majors. This view is consistent with discussions with students enrolled in the Winter Quarter 2000 course.

![Figure 1. Precalculus Enrolment by Year](image-url)
As there are substantial differences between the student profile depending on the quarter, I will focus this report on Winter Quarter data. For example, the Winter Quarter 2000 course consisted in 64% first year students, in contrast to a typical Fall Quarter enrolment of more than 80%, c.f. Figure 1. Approximately one-third of the students has not declared a major. The three largest majors at UCSB, Biological Science, Business Economics and, Psychology, are the dominant majors in the class, c.f. Figure 2. While a significant number, about one-third, of these students report having a positive attitudes about mathematics, this is not the case for most of them. On the average, Math 15 students are ambivalent about mathematics as reflected in
an average of 3.01 on a six point scale, c.f. Figure 3. Despite the title ‘Precalculus,’ a Fall 1996 Mathematics Department (UCSB 1996) study of students who have taken Math 15 from 1991 until Winter 1996 showed that about 31% of the students took no further mathematics classes, 12% repeated the course, 27% enrolled in the social and life science calculus course and, 28% enrolled in the physical science and engineering calculus course. Students may repeat a course if they have obtained a grade of C- or lower. The final grade distribution suggests that, despite the elementary level of the course, few students attain a mastery of the material in the view of their instructors and many fail to attain a grade of at least a C, c.f. Figure 4.

![Bar graph showing course grades for Fall 1996](image)

*Figure 4. Fall 1996 Precalculus Course Grades*

The Fall 1996 grades provide evidence of a context consistent with that of the 1996 Mathematics Department Study.

3. COURSE GOALS FOR WINTER 2000

The overarching goal for Winter 2000 was to increase the overall level of student performance as measured by the course grades and performance on the calculus gateway examination. Since this is the ‘terminal mathematics course’ for about a third of the enrolled students, the secondary course goals were broader than simply preparing students for the physical sciences calculus course. While the official course description is "A functional approach integrating algebra and trigonometry. Topics include: one-to-one and onto functions; inverse functions; properties and graphs of polynomial, rational, exponential, and logarithmic functions; properties and graphs of inverse trigonometric identities; and trigonometric equations", the secondary goals were selected with the broader needs of students in mind. The first of these goals was to expand the range of student mathematical work or activities to include reading the text (many students appear to rarely or unproductively read
Making large lectures effective

Mathematics texts) and to devote more productive time and effort to working on learning and doing mathematics, either alone or with other students. The second goal was to expand the students' understanding of mathematics from a collection of facts and procedures to be memorized and applied in familiar contexts, to an understanding of mathematics in which thinking played the primary role. One specific manifestation of this goal would be a productive reaction to opportunities to apply concepts and methods in new settings. The third goal was to improve the students' attitudes about mathematics and their ability to 'do mathematics.'

4. WINTER 2000 STRATEGIES

An excellent basic resource on teaching at the college and university level is Steven Krantz's book, How to Teach Mathematics (Krantz, 1999). In this book, Krantz provides solid practical advice about the art of talking to a large group of students in a manner that will not alienate too many of them and, perhaps, even make it a lively experience for at least some of them. But, he says "There are virtually no data to support the contention that small classes are 'better' than large classes for mathematics teaching. Statistics do not indicate that students in small classes perform better or retain more. The statistics do indicate that students feel better about themselves and the class, and enjoy the situation much more if the class is small. Another way to say this is that large lectures are not a good tool for engaging students in the learning process!" One recommendation, of a practical nature, is to have every detail prepared in advance so that nothing disrupts the smooth flow of the class meeting or examination. In short, minimize the unexpected, at least from the perspective of the student. Krantz asks, "What is it that makes teaching a large class seem to be difficult? Is it the size of the room, the populations of students, the use of a microphone, the feeling that there is less room for error, or what? What can we as math teachers - not preachers or performers - do to make a large class still seem like a family? What can we do to make a large class still feel like a place where learning is taking place?"

Among those contributing appendices to the book are proponents or defenders of large classes, those who see themselves as 'sages-on-the-stage.' Frequently advocates for the model of 'direct instruction' argue that the true cause of a students' failure lies with the student and not with the teaching. For example, George Andrews (1999), states that "students are not studying enough" and outlines several factors the he believes contributes to this situation: a lack of concern for this in reform efforts, a lack of concern for the amount of time devoted to homework, large lectures, computerized student evaluations, limited importance of teaching in promotion, and a lack of support for 'high standards.' "Too plentiful student success" represents, he says, a failure to "maintain standards." In another appendix, Richard Askey (1999), discusses problems associated with the use of graduate student assistants, their role in the instruction. He also affirms the necessity of 'maintaining standards' but offers no advice on ways to insure that effective teaching occurs. H. Wu (1999), another avowed 'sage on the stage,' states in his appendix that "There are situations where lectures are very effective, and in fact,
there are even circumstances that make this method of instruction mandatory." How
Wu proposes to measure effectiveness and against what standard it is measured is
left unstated. His focus is on the role of the teacher in the implementation of a
'contract' with the student. "Lecturing is one way to implement this contract. It is an
efficient way for the professor to dictate the pace and to convey his vision to the
students, on the condition that students would do their share of groping and
staggering toward the goal on their own." While acknowledging that "Lecturing can
take many forms," Wu provides no alternative to the 'traditional lecture.' One
benefit of the lecture approach, he tells us, is that instructors can share ideas and
insights not available in the text. "I see no reason why students cannot profit from
such insight by making an effort to understand the lectures instead of adamantly
resisting them by coming to class unprepared, or for that matter, leaving it without
making an effort to understand it later."

An assumption underlying the praise of the traditional lecture is that the cause of
any 'failure' lies principally with the student. The sole responsibility of the teacher
is the 'transmission of information.' I do not agree. Too often the nostalgic paens
for the 'good old days' ignore the widespread failure of large numbers of students to
become mathematically proficient. They wish for a time when professors were
treated like divinity, were not expected to be sensitive to insulting, offending or,
otherwise alienating students, especially women or members of minority groups;
when 'good lectures' meant only inspiring or entertaining repetitions of the material
found in the text. Some, typically the textbook authors themselves, would go so far
as to recite the 'jokes' that were included in the texts. Another common feature is
the attitude that only the very few, persons such as themselves, were capable or
worthy of success common among college and university faculty members. And,
indeed, this opinion of students is still found across the entire K-16 educational
landscape. Such thinking is not, I believe, compatible with making student
achievement the principal measure of success of the course. Therefore, a more
careful analysis of the large class setting is required, whether or not, a traditional
lecture is the instructional method that is ultimately selected.

With few exceptions, the students come to Math 15 with a history of limited
mathematical success, with little appreciation of their on ability to learn or do
mathematics, without successful strategies for learning mathematics and, with
'passing the course' as their principal goal. Few recognize the need to change their
approach to learning, even with a record of previous failure. Nor do they seek an
understanding of mathematics sufficient to allow them to apply its concepts and
methods in new settings. In order to place increased focus on the student, I decided
to assign homework and grade on a weekly basis. They are passive listeners. While,
perhaps, physically present during class meetings, they come unprepared to learn or
unprepared to participate actively in the class. As a consequence, the structure of the
class and its meetings needed to address these facts in order to make learning
feasible. In order to place greater focus on student performance, I decided to assign
and grade homework problems on a weekly basis. Graded by an undergraduate
assistant, the homework score was combined with scores from 'surprise quizzes' to
give 25% of the comprehensive course grade. There were two course exams (25%) and
a comprehensive final exam (50%) that also formed portions of the
comprehensive course grade. Students who achieved a 'physical calculus qualifying grade' on the gateway exam before the end of the quarter received a bonus award of 15 points (out of a scaled score of 100) added to the comprehensive score. Finally, students were told prior to the comprehensive final examination that their course grade would be no lower than the grade received on that final examination. The later option was instituted in order to encourage sustained effort for those who may have faltered on some earlier aspect of the course.

The text for the course was Functions Modeling Change: A Preparation for Calculus by Connally, Hughes-Hallett, Gleason, et. al., John Wiley and Sons, Inc., 2000. It was selected in order to promote a larger understanding of mathematics, its concepts, its methods and, its application both to calculus and more widely. During the ten-week quarter, reading assignments and problems were taken from the first nine chapters, with topics on rational functions only briefly discussed during the last week of the term. Some supplementary material was added, mostly of a historical nature. Students were encouraged, but not required, to use graphing calculators in all aspects of the course. While some of the assigned homework problems required the use of a calculator, the course examinations were constructed so as to be calculator neutral. Furthermore, some questions explicitly forbid the use of calculators.

Students attended 150 minutes of 'lecture' and 50 minutes of mandatory 'discussion section' with a graduate teaching assistant designated by the department. The discussion meetings focused on returning graded homework and answering a few questions on the course material. Some students elected to join the Achievement Program’s academic workshops sponsored by the California Alliance for Minority Participation. These workshops met 150 minutes each week. They included specific attention to the needs of individual students such as an informal assessment of their preparation for the course, their study skills or approaches to reading the text as part of the learning process. The workshops were staffed by a graduate student and a highly successful undergraduate. They both attended weekly staff meetings at least partially devoted to increasing the effectiveness of the workshops through diagnosis of individual student progress, promotion of productive mathematics based interactions among students, and in-depth review of returned homework, quizzes, and supplementary work associated with the workshop groups.

In response to prior student experience and attitude, the seventy-five minute 'lecture' did not follow the traditional formula but employed a format that was primarily 'interactive' in style:

• the first segment consisted of a brief administrative reminder of assignment and exam content and dates followed by an invitation for questions from students;
• the second segment was devoted to a discussion of any systematic mathematical problems that had been identified by course assistants or the instructor;
• the third segment was frequently, but not always, launched with a group 'surprise quiz' consisting of a question taken from the reading assigned for preparation for the day's lecture. These 'quizzes' produced written responses from groups of students, typically two or three. These were given to the instructor following a plenary discussion and student presentations of various alternative approaches to the question, including any underlying concepts and strategies or algorithms employed in the response. This discussion was at the 20
minute mark (of the 75 minute lecture period) and represented a bit of a 'change
of pace' marking the move from old to new ideas or methods;

- the fourth and longest segment concerned new concepts and/or methods. They
grew out of the problem or the surprise quiz and were developed in a manner
based on the subsequent discussion. No more than one or two new key elements
formed the focus of the work;

- time permitting, any secondary new material was incorporated in the
presentation and discussion of a closing question/problem situation;

- in closing, students were reminded of homework, quizzes, material to be
reviewed, etc.

Beginning with the first meeting and throughout the ten weeks of the quarter, I
recognized an increasing number of students by name during the lecture. In order to
effectively shrink the apparent size of the class, an effort was made to identify
students sitting in all areas of the lecture hall, specifically those sitting in the very
last rows. As students typically are very consistent in when and with whom they sit,
a mental map can be developed and used quite reliably. Those students who ask
questions are requested to provide their names, which are then used in the course of
responding to their question. Each student is thanked for the question and is
provided with explicit appreciation of the usefulness of the question in addressing
course goals. For example, in an attempt to begin encouraging students to interact
mathematically with their classmates, I asked students to find a partner to whom
they were to describe a 'real life' example of a function that they had encountered
within the last day. After about four or five minutes of conversation, I selected a
student from the middle of the class and asked them their name and major. Using
their name, I asked them to introduce me to their partner. Most often they do not
know or recall the name and so I ask everyone to take another few minutes to
introduce themselves to the persons seated on each side of them. Once that is done,
using their name, I ask the first person selected to describe the example of their
partner's function. Rarely can they do this. If this is the case, I ask them to repeat the
conversation with their partner and to be prepared to not only describe the example
but explain why it is an example. Finally I ask about five students to describe their
partner's example and to compare it with their own. This exercise requires a fair bit
of time but is used to lead into an explanation of what I am seeking from them by
way of interaction with me during the class meeting. Specifically, I will be often
telling them about some mathematics or answering a question posed by another
student. I expect that they should be able to explain what I have said before we agree
to move on to another matter. If not, they are responsible for asking questions or for
ensuring that their questions are answered at another time. Failure to follow through
on this is a significant contributing obstacle to student learning and requires
reminders regularly through out the course.

Another example of my effort to shrink the apparent class size occurs during the
'surprise quizzes.' I regularly walked about the lecture hall and engaged groups of
students, by name, in discussions about the question. I am seeking information as to
their preparation of the material and to identify aspects of the material about which
there may be questions shared by large numbers of students. This information was
used to evaluate and to revise the lecture material for the day. As is often the case,
students have not prepared for the meeting by reading the assigned material. I remind them that not doing so impacts their course grade through the quiz grade and the reduced usefulness of our time together. Attending class means coming prepared to do the work of the day!

Three ‘office hours’ were scheduled each week by my graduate assistant and myself. Few students actually made use of this opportunity to discuss the course work. Most such questions arose during the lecture or discussion session or immediately following them. A substantial amount of interaction occurred via email, most of which could be characterized as non-mathematical: classes missed due to personal problems; requests to submit ‘late’ homework, requests for ‘make-up-exams’, or complaints about the effectiveness of the graduate assistants and the accuracy of the grading on homework and exams. As a matter of policy, no ‘late’ homework was graded and ‘make-up-exams’ were not given. I did agree to make a notation in my grade record that homework was done by students even though it was not graded. This was an effort to keep them productively engaged in the class and doing the homework, even if late. These notations did not, ultimately, have any influence on the course grades as the involved students had grades that were quite clearly within each final grade cohort.

All individual pieces of course work were evaluated on a six point scale with six being assigned to technically correct and fully explained mathematical work and zero assigned to work that provided no contribution to progress on the associated problem or question. Five corresponded to minor technical errors or slight weaknesses in explanations for which no further mathematical discussion would be necessary in order that a student revision be fully correct. A four was associated to work with somewhat more important errors, mathematical gaps or missing elements of an explanation for which only a modest discussion or reminder of some aspect of the work should be adequate to support a successful revision. A three corresponded to a submission for which some important aspect was absent or inadequate while others were present. In the case of a three, some additional mathematical learning might be required before a complete solution could be provided. Two indicated that some productive work and justification was provided, but that much was missing and further instruction was clearly needed. A score of one indicated that there was some potentially productive work provided but only a very modest amount with principal elements missing. This grading scheme caused some confusion for the graduate assistants, the undergraduate grader, and the students enrolled in the course. As a consequence, the evaluation of student work required a much more careful reading and, as a consequence, the reading required more training and supervision by the instructor to ensure an acceptable quality of evaluation. While students seemed to welcome the ‘partial credit’, they were quite unaccustomed with the requirement that each answer be supported by an explanation in order to receive full credit. Technically correct calculations and numerical answers with absent explanations would receive a maximum score of three.

Finally, much more and different information about the student experience and performance was collected than is the custom. I did this in order to begin a systematic effort to improve the effectiveness of the course. This report represents one dimension of that effort.
Figure 5. Examination I Grades by Class Rank

The distributions of scores on homework, course exams and, comprehensive exams are shown in Figures 5 - 9. One dimension of overall student performance is represented by average scores of 3.690 on Exam I, Figure 5, 2.205 on Exam II, 3.646, Figure 6, on Homework/Quiz, Figure 7, 2.759 on the Final Exam, Figure 8,

Figure 6. Examination II Grades by Class Rank
and 2.744 on the Course Composite grade, Figure 9. These data suggest that the learning of major mathematical components of the course was not demonstrated by the performance of students on the final exam. Indeed, the second examination
Figure 9. Composite Grade by Class Rank

Figure 10. Homework Grade versus Final Examination Grade
performance, covering more complex material than that found on the first exam, foreshadows this fact. The average final course grade was 2.593, on a four point scale, compared to the 2.45 recorded in Winter Quarter 1999. In the absence of concrete achievement standards for the course, it is not possible to determine the extent to which the increased average grade is evidence of actual increased student achievement or reflects different expectations among the instructors.

5. ANALYSIS

One of the important questions that can be asked about the meaning of this data concerns the value of collecting and grading homework. For me, the purpose of graded homework is to give value to work designed to promote student learning of the concepts and methods. Visually and numerically, as represented by a covariance of 0.175 and an R-Square of 0.0263, Figure 10, there is strong evidence that the homework did not make a significant contribution to student performance on the comprehensive final examination. One could conclude, therefore, that the homework component did not make a measurable contribution to student learning and, thus, to the achievement of the course goals.

![Figure 11. Final Grade Comparison between Winter 1999 and Winter 2000](image)

Another innovation was the offer of 'bonus' credit for passing the calculus gateway examination. Historically, 28% of the students taking Precalculus enroll in the physical science calculus course. In Winter 2000, 32.7% of the Precalculus students achieved the admission standard on the gateway examination necessary for enrollment in this course. Of these, 22 received A's, 11 B+'s, 4 B-'s and 4 C+'s, Figure 11. Of these 41 students, roughly one-third of the final course enrollment, seven students, or 5.7% of the class, received an improved grade as a consequence of the bonus. One interpretation is that the effort dedicated to passing the gateway...
examination also contributed to student success on the comprehensive examination despite the multiple choice structure of the latter and the requirement for explanations in the former.

Several consistent themes arose during the course of interactions with students enrolled in the course. They are (1) a desire that the course be graded ‘on a curve’; (2) the opinion that asking for solutions to ‘unfamiliar’ questions was not appropriate; (3) a confusion between the memorization of formulae and methods and ‘understanding’ them; and (4) the belief that course grades should be based on effort rather than performance. In interviews with students the goal of working problems and studying for the course in order to make it possible to solve problems without ‘having to think’ arose frequently. Mathematical accomplishment, it appeared, was not a question of being able to use the concepts and methods to figure out how to solve a problem but, rather, a question of being able to recognize the problem type and apply the solution procedure appropriate to the question. In looking at student work on examinations, it seems likely that a careful reading of the problem statement and the question is not occurring. Frequently, students would state that they understood how to solve the problem but were unable to explain either the problem setting, the question, or their proposed solution method. Indeed, when asked about this, students frequently apologized for their mathematical ineptitude by claiming they were not ‘mathematical’ persons or by blaming their prior mathematical courses. In the latter case these courses ranged from those of a hierarchically structured curriculum with a focus on algorithmic practice to those that integrated areas of mathematics and its applications. In common was the statement that “I never understood what I was doing.”

6. CONCLUSIONS CONCERNING NEXT STEPS

Despite an intensive effort to actively engage all students in productive work on the curriculum, many of the students remained on the sidelines unconcerned gaining the expected understanding of the mathematics. Preparation for class meeting was inadequate and, even though physically present, class participation in the work was uneven. One possible strategy to address this problem would be to increase the contribution of the ‘surprise quizzes’ to the course grade. Greater emphasis could be placed on the individual responsibility to read the material in advance. Some instructors, not in mathematics, have made this preparation a requirement of class attendance. While this seems a bit extreme, it does make clear the student responsibility to participate actively in the course. I will certainly devote more time to addressing the lack of understanding of what learning mathematics will mean in the course and to suggesting strategies that will help students learn this mathematics successfully.

In particular, I will try to more clearly describe the purpose of collecting and grading homework in relationship to the overall course goal. In short, the purpose of the homework is not to “do the homework, but to understand the math involved.” I will also arrange meetings with the graduate students and the various campus agencies that provide tutoring to help them understand ways in which they can better
help students in my course be more successful. Another dimension of this is the need to have a better understanding of the character of the standards used in the assessment of student work. As I had done for calculus courses earlier, I will develop a small collection of problems and a variety of 'solutions' together with evaluations and explanations of these evaluations. These will help students, tutors, graders, and graduate students have a more concrete understanding of the mathematical performance sought in the course. Finally, as I have in other classes, I will replace office hours by weekly hours in a 'mathematics laboratory' during which time each student will be required to personally give me their homework. I will scan a selected problem to assess the quality of the work and give the student a quick assessment of the acceptability of their submission. In the event that it is below the standard, I will point out difficulties and invite them to immediately revise their submission. This strategy has worked well in calculus classes in the past by leading to greater individual responsibility for the quality of the work and, consequently, higher levels of overall class success.

Working on knowing individual students did seem to promote more open interaction and questions. It is uncertain if this had the desired 'halo effect' of shrinking the apparent class size. Extending the interaction to regularly include each student, may help address this aspect. I will keep closer records of interaction and of student performance next year in order to gain a clearer understanding of the impact of this feature of the course.

The successful teaching of Precalculus is, I believe, a very challenging task if one is willing to accept the notion that success is measured by the mathematics learned by the students. I have tried to illustrate, in the description of several aspects of my effort, how it is possible to teach large classes at a college or university level in a manner that is student centred. I have not gone into my analysis of how students fared on more specific mathematical tasks, such as the use of inverse functions in the context of trigonometric and exponential/logarithmic functions, which involves an item analysis of problems on examinations. Rather, I have tried to give a larger grain overview of key strands of how I approach this task so as to be able to systematically improve the effectiveness of the class. In doing so, I encourage you to join in the effort to move the spotlight from the 'sage-on-the-stage' to the 'students-in-the-class.' This is where our successes and our failures to teach are really determined.

REFERENCES


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