Name: Key

Mathematics 108A: Practice Quiz G

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Part I. True-False. Circle the best answer to each of the following questions.

1. The \( m \times n \) matrix \( A \in M_{m \times n}(\mathbb{R}) \) defines a linear map

\[
L_A : \mathbb{R}^n \to \mathbb{R}^m \quad \text{by} \quad L_A(x) = Ax
\]

The null space of this linear map is the space of solutions to the homogeneous linear system

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= 0, \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= 0, \\
    &\vdots
    \end{align*}
\]

\[
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0
\]  \hspace{1cm} (1)

\(\fbox{TRUE}\)

2. The range of the linear map \( L_A \) is the space of vectors \( b = (b_1, \ldots, b_m) \) in \( \mathbb{R}^m \) such that the linear system

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1, \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2, \\
    &\vdots
    \end{align*}
\]

\[
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m
\]

has a solution.

\(\fbox{TRUE}\)

\(\fbox{FALSE}\)

3. According to one of the Main Theorems of Chapter 2 in the text,

\[
\dim(N(L_A)) + \dim(R(L_A)) = m.
\]

\(\fbox{TRUE}\)

\(\fbox{FALSE}\)
4. Suppose that \( m = n \). Then the nonhomogeneous linear system
\[

gives a solution for any choice of \((b_1, b_2, \ldots, b_n)\) if and only if the corresponding homogeneous linear system
\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= 0, \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= 0, \\
    \cdots & \cdots \cdots \\
    a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= 0
\end{align*}
\]
has no solutions.

\( \text{TRUE} \) \hspace{1cm} \text{FALSE} \)

**Part II. Multiple Choice.** Circle the best answer to each of the following questions.

5. Suppose that \( L_A : \mathbb{R}^5 \to \mathbb{R}^3 \) be the linear transformation defined by \( L_A(x) = Ax \), where
\[
\begin{pmatrix}
    1 & 7 & 0 & 2 & 1 \\
    0 & 0 & 1 & 4 & 3 \\
    0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
Then the dimension of \( N(L_A) \) is
\[ \text{a. 1} \hspace{1cm} \text{b. 2} \hspace{1cm} \text{c. 3} \hspace{1cm} \text{d. 4} \hspace{1cm} \text{e. None of these} \]

6. On the other hand, the dimension of \( R(L_A) \) is
\[ \text{a. 1} \hspace{1cm} \text{b. 2} \hspace{1cm} \text{c. 3} \hspace{1cm} \text{d. 4} \hspace{1cm} \text{e. None of these} \]

7. If \( L : \mathbb{R}^5 \to \mathbb{R}^3 \) is ANY linear transformation such that the dimension of \( R(L) \) is three, then the dimension of \( N(L) \) is
\[ \text{a. 1} \hspace{1cm} \text{b. 2} \hspace{1cm} \text{c. 3} \hspace{1cm} \text{d. 4} \hspace{1cm} \text{e. None of these} \]

8. If \( L : \mathbb{R}^4 \to \mathbb{R}^4 \) is ANY linear transformation such that \( N(L) = \{0\} \), then the dimension of \( R(L) \) is
\[ \text{a. 1} \hspace{1cm} \text{b. 2} \hspace{1cm} \text{c. 3} \hspace{1cm} \text{d. 4} \hspace{1cm} \text{e. None of these} \]

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