True-False. Circle the best answer to each of the following questions. Each question is worth 2 points.

1. If

\[ W_1 = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 = 0\}, \quad \text{and} \quad W_2 = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 = 3x_1\}, \]

then \( \mathbb{R}^2 \) is the direct sum of \( W_1 \) and \( W_2 \).

- [ ] TRUE
- [X] FALSE

2. If

\[ W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 - 2x_3 = 0\}, \quad \text{and} \quad W_2 = \text{Span}(1, 1, 1), \]

then \( \mathbb{R}^3 \) is the direct sum of \( W_1 \) and \( W_2 \).

- [X] TRUE
- [ ] FALSE

3. Regard \( V = M_{2,2}(\mathbb{C}) \) as a vector space over \( \mathbb{C} \), and let

\[ U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad I = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \]

\[ J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad K = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \]

Then the list \( (U, I, J, K) \) is a basis for the subspace

\[ \mathbb{H} = \{ A \in M_{2,2}(\mathbb{C}) : A = tU + xI + yJ + zK, \text{ where } t, x, y, z \in \mathbb{R} \}. \]

- [ ] TRUE
- [X] FALSE

4. If addition is vector addition and multiplication is matrix multiplication, then \( \mathbb{H} \) satisfies all of the field axioms except for the commutative law

- [X] TRUE
- [ ] FALSE
5. Let $W$ be the subspace of $\mathbb{R}^4$ defined by

$$W = \text{span}((1,2,0,5),(0,0,1,4))$$

Then $((1,2,0,5),(0,0,1,4))$ is a basis for $W$.

TRUE \hspace{2cm} FALSE

6. Let $W^\perp$ be the orthogonal complement to the space $W$ considered in the preceding problem. Then $((-2,1,0,0),(-4,0,-5,1))$ is a basis for $W^\perp$.

TRUE \hspace{1cm} FALSE

7. Let $L_A : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ be the linear maps defined by

$$L_A(x) = Ax \text{, where } A = \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

Then $((-2,1,0,0),(-4,0,-5,1))$ is a basis for $N(T)$.

TRUE \hspace{2cm} FALSE

8. Let $L_A : \mathbb{R}^2 \rightarrow \mathbb{R}^4$ be the linear maps defined by

$$L_A(x) = Ax \text{, where } A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \\ 5 & 4 \end{pmatrix}$$

Then $((1,2,0,5),(0,0,1,4))$ is a basis for $R(T)$.

TRUE \hspace{2cm} FALSE