Name: Key

Mathematics 108A: Quiz 3
Friday, May 28, 2010
Professor J. Douglas Moore

Part I. True-False. Circle the best answer to each of the following questions. Each question is worth 2 points.

1. The set $Z_4 = \{0, 1, 2, 3\}$, with the operations of addition and multiplication modulo four is a field with finitely many elements

   **TRUE**

2. If $W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + 2x_2 - 3x_3 = 0\}$ and $W_2 = \text{Span}(1,1,1)$, then $\mathbb{R}^3$ is the direct sum of $W_1$ and $W_2$.

   **TRUE**

3. Let $\beta = (v_1, v_2)$ be the basis for $\mathbb{R}^2$ defined by

   $v_1 = \left(\begin{array}{c}1 \\ 2\end{array}\right), \quad v_2 = \left(\begin{array}{c}1 \\ 3\end{array}\right)$

   If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map such that $T(v_1) = 3v_1$ and $T(v_2) = 7v_2$, then the matrix of $T$ with respect to $\beta$ is

   $[T]_{\beta}^\beta = \left(\begin{array}{cc}3 & 0 \\ 0 & 7\end{array}\right)$.

   **FALSE**

4. An element $A \in M_{n \times n}(\mathbb{F})$ is invertible if and only if it is a product of elementary matrices.

   **TRUE**

5. Let $W$ be the subspace of $\mathbb{R}^4$ defined by

   $W = \text{span}((1, 2, 0, 5), (0, 0, 1, 4))$

   If $W^\perp$ is the orthogonal complement of $W$, then $((1, 2, 0, 5), (0, 0, 1, 4))$ is a basis for $W^\perp$.

   **FALSE**
Part II. Give complete proofs of each of the following statements

1. (5 points) Find the inverse of the matrix

\[
A = \begin{pmatrix}
1 & 3 & 0 \\
1 & 4 & 0 \\
0 & 2 & 0
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
1 & 3 & 0 \\
0 & 1 & 0 \\
0 & 2 & 0
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
1 & 3 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[
A^{-1} = \begin{pmatrix}
4 & -3 & 0 \\
-1 & 1 & 0 \\
0 & 0 & \frac{1}{2}
\end{pmatrix}
\]

2. (5 points) Find the determinant of the matrix

\[
A = \begin{pmatrix}
2 & 1 & 0 & 0 \\
1 & 2 & 1 & 0 \\
0 & 1 & 2 & 1 \\
0 & 0 & 1 & 2
\end{pmatrix}
\]

(Hint: It is easiest to use elementary row operations)

\[
2 \begin{pmatrix}
\frac{1}{2} \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
1 & \frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{1}{2} & 0 \\
0 & 0 & 1 & \frac{1}{2} \\
0 & 0 & 0 & 1
\end{pmatrix}
= 3 \begin{pmatrix}
1 & \frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{1}{2} & 0 \\
0 & 0 & 1 & \frac{1}{2} \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
4 \begin{pmatrix}
1 & \frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{1}{2} & 0 \\
0 & 0 & 1 & \frac{1}{2} \\
0 & 0 & 0 & 1
\end{pmatrix}
= 5
\]