Math 3C Lecture 1

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Thomas Robert Malthus (1766-1834) published various editions of his famous treatise

An essay on the principle of population, between 1798 and 1826, in which he claimed that “the power of population is indefinitely greater than the power of the earth to produce subsistence for man.”

In modern terms, one might phrase his argument thus: Since the planet has finite resources, unless the birth rate is kept down through “moral restraint,” the death rate must go up due to famine, disease and/or war.
Famine seems to be the last, the most dreadful resource of nature. The power of population is so superior to the power in the earth to produce subsistence for man that premature death must in some shape or other visit the human race. The vices of mankind are active ministers of destruction, and often finish the dreadful work themselves. But should they fail in this war of extermination, sickly seasons, epidemics, pestilence, and plague advance in terrific array, and sweep off their thousands and ten thousands. Should success be still incomplete, gigantic inevitable famine stalks in the rear, and with one mighty blow, levels the population with food of the world. (Malthus, An Essay on the Principle of Population, reprinted by Univ. of Michigan Press, Ann Arbor, Mich., 1959, p.49.)
Let

\[ y(t) = \text{population at time } t. \]

Imagine that population \( y \) is measured in millions or billions of people and time \( t \) is measured in years.

We think of \( y(t) \) as a \textbf{function} of \( t \).

We want to determine what this function is \textbf{under given assumptions}.

Let us make the assumptions that the birth rate and death rate are constant and that there is no immigration or emigration.
Then the instantaneous rate of change in the number of people per unit time should be proportional to the number of people.

The instantaneous rate of change is represented by the derivative

\[
\frac{dy}{dt}
\]
denote the rate of change of population with respect to time.

So the above sentence translates into the Malthus model for population growth:

\[
\frac{dy}{dt} = ky,
\]
where \( k \) is a constant of proportionality called the **instantaneous growth rate**.
The equation

$$\frac{dy}{dt} = k y$$

is called the equation of exponential growth (when $k > 0$) or decay (when $k < 0$). It is a simple and important example of a **differential equation**.

Solutions to differential equations are **functions**, not numbers. To solve the equation of exponential growth or decay, we need to find a function $y(t)$ such that

$$\frac{d}{dt}(y(t)) = ky(t).$$
Rigorous thinking: $y(t)$ is a function of $t$ and

$$\frac{d}{dt}(y(t))$$

is its derivative.

Intuitive thinking: $y$ is a variable which depends on the variable $t$. $dy$ is the change in $y$ that corresponds to a change $dt$ in $t$.

Both points of view are useful in solving problems.
Using the second point of view we can separate variables and obtain

\[ \frac{dy}{y} = kdt. \]

We can then integrate both sides to solve the differential equation.
The **solutions** to the differential equation
\[
\frac{dy}{dt} = ky
\]
are the **functions**
\[
y(t) = ce^t,
\]
where \( c \) is a constant called the constant of integration.

If we set \( t = 0 \), we find that
\[
y(0) = ce^0 = c.
\]
Thus \( c \) is the initial population.
Assume that the current world population is $6.7 \times 10^9$.

Area of the earth in square feet

$$A = 4\pi(4000)^2(5280)^2 = 5.6 \times 10^{15}.$$  

Suppose the population grows at a rate $k = .02$ (two percent per year). How many years does it take until there is one person per square foot?

Need to solve

$$5.6 \times 10^{15} = e^{0.02t}(6.7 \times 10^9),$$

$$e^{0.02t} = \frac{5.6}{6.7} \times 10^6,$$

$$0.02t = \log \left( \frac{5.6}{6.7} \times 10^6 \right),$$

$$t = 50 \log \left( \frac{5.6}{6.7} \times 10^6 \right).$$