Homework is available from WebWork:

http://webwork.math.ucsb.edu/

Your username and password are both set to be your perm number. (You can reset your password if you want.)

Usually homework will be due on Fridays, but the first assignment (which is rather short) is due Monday, April 6.
Last time we considered the Malthus law for population growth, in a given country, for example. Let

\[ y(t) = \text{population at time } t. \]

Imagine that population \( y \) is measured in millions of people and time \( t \) is measured in years.

We want to determine what this function is under certain assumptions: We assume that birth and death rates are constant and that there is no immigration or emigration.

This implies that the rate of change of population with respect to time is proportional to population. We can express this sentence as a differential equation.
The Malthus model for population growth is:
\[ \frac{dy}{dt} = ky, \]
where \( k \) is a constant, called the \textit{instantaneous growth rate}. The Malthus model provides a simple and important example of a \textit{differential equation}, that is, an equation which involves derivatives.

To solve it, we can \textit{separate variables} to obtain
\[ \frac{dy}{y} = k dt, \]
and then integrate both sides.
We find that the solutions to the differential equation
\[ \frac{dy}{dt} = ky \]
are the functions
\[ y(t) = ce^{kt} \]
where \( c \) is a constant called the constant of integration.

Since
\[ y(0) = ce^0 = c \]
we find that in the Malthus model for population growth \( c \) is the population at time \( t = 0 \).
After one year has elapsed the population is $y(1) = ce^k$.

Thus the increase in population after one year has elapsed is $ce^k - c = c(e^k - 1)$.

The percentage increase in one year is $e^k - 1$.

$$e^k = 1 + k + \frac{k^2}{2!} + \frac{k^3}{3!} + \cdots$$

If $k$ is small, for example $k = .01$, then $e^k - 1 \approx k$.

Thus $k = .01$ represents an approximate growth rate of 1% per year.
The population of Afghanistan is growing at the rate of 2.63% per year. What is the instantaneous growth rate?

If \( c \) is the population at the beginning of a given year, the population at the beginning of the following year will be \( ce^k \), so

\[
1.0263c = ce^k, \quad e^k = 1.0263,
\]

and the \textbf{instantaneous} growth rate is

\[
k = \log(1.0263) = .0260.
\]

(Note that \( \log(1+k) \) is approximately equal to \( k \) when \( k \) is small.)
## Population Growth Rates for Selected Countries:

<table>
<thead>
<tr>
<th>Country</th>
<th>Percentage growth</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaza Strip</td>
<td>3.61%</td>
<td>.0355</td>
</tr>
<tr>
<td>Somalia</td>
<td>2.82%</td>
<td>.0278</td>
</tr>
<tr>
<td>Afghanistan</td>
<td>2.63%</td>
<td>.0260</td>
</tr>
<tr>
<td>Iraq</td>
<td>2.56%</td>
<td>.0253</td>
</tr>
<tr>
<td>Haiti</td>
<td>2.49%</td>
<td>.0246</td>
</tr>
<tr>
<td>Egypt</td>
<td>1.68%</td>
<td>.0167</td>
</tr>
<tr>
<td>India</td>
<td>1.58%</td>
<td>.0157</td>
</tr>
<tr>
<td>USA</td>
<td>.88%</td>
<td>.0088</td>
</tr>
<tr>
<td>China</td>
<td>.63%</td>
<td>.0063</td>
</tr>
<tr>
<td>UK</td>
<td>.28%</td>
<td>.0028</td>
</tr>
<tr>
<td>Japan</td>
<td>.14%</td>
<td>.0014</td>
</tr>
<tr>
<td>Germany</td>
<td>.04%</td>
<td>.0004</td>
</tr>
</tbody>
</table>

www.indexmundi.com
Assuming a constant instantaneous growth rate \( k \), what is the population in 20 years?

If \( c \) is the current population, the population in 20 years is \( ce^{20k} \).

Population of China is \( 1.330044 \times 10^9 \). Instantaneous growth rate for China is \(.0063 \).

\[
\begin{align*}
\text{if } c &= 1.330044 \times 10^9 \text{ and } k = .0063, \text{ then} \\
1.50865
\end{align*}
\]

Population of India is \( 1.147995 \times 10^9 \). Instantaneous growth rate for India is \(.0157 \).

\[
\begin{align*}
\text{if } c &= 1.147995 \times 10^9 \text{ and } k = .0157, \text{ then} \\
1.57148
\end{align*}
\]
If the population \( y(t) \) is given by the formula

\[ y(t) = ce^{kt}, \]

how long does it take population to double?

The initial population is \( c \) and the population after \( t \) years have elapsed is \( ce^{kt} \). So we want to find \( t \) such that

\[ 2c = ce^{kt} \quad \text{or} \quad 2 = e^{kt}. \]

Thus

\[ \log 2 = kt \quad \text{or} \quad t = \frac{\log 2}{k}. \]
Thus we conclude that
\[ \text{doubling time} = \frac{\log 2}{k}, \]
\[ \text{doubling time} = \frac{.693147\ldots}{k}, \]
approximate doubling time = \frac{.7}{k}.

World population is growing at about 1.188% per year which corresponds to \( k = .01181 \).

Doubling time for population of the earth:
\[ \text{doubling time} = \frac{\log 2}{.01181} = 58.6915. \]