Math 240B: Problem Set VI

March 6, 2009

Due: Friday, March 13, 2009.

Exercise X. a. Show that in Minkowski space-time $\mathbb{L}^4$, 

$$\star \star = (-1)^{k+1} : \Omega^k(\mathbb{L}^4) \longrightarrow \Omega^k(\mathbb{L}^4).$$

b. Determine $\star dt$, $\star dx$, $\star dy$ and $\star dz$.

c. Show that Maxwell’s equations can be expressed in terms of the Faraday tensor (described in the lecture notes) as

$$dF = 0, \quad d(\star F) = \star (4\pi J),$$

where $J = -\rho dt + J_x dx + J_y dy + J_z dz$.

Exercise XI. Suppose that $M = S^2$, the standard unit two-sphere in $\mathbb{E}^3$, with Riemannian metric expressed in spherical coordinates as

$$\langle \cdot, \cdot \rangle = (\sin^2 \phi) d\theta \otimes d\theta + d\phi \otimes d\phi.$$

Determine the Laplace operator on functions in this case.