Practice Problems For Final

Reminders about the final exam

- The final will be on Thursday, March 17 from 8-11am in your regular classroom
- Bring a photo ID to the exam
- There are NO calculators allowed
- You are allowed a $4 \times 6$ notecard with whatever you want written on both sides
- The final is worth 50% of your grade
- The final is cumulative (Chapters 1 - 7.4)

Disclaimer: The following problems and formulas are based on what I believe to be important, which may or may not coincide with what will be covered on your final exam. This document should in no way replace careful studying of your notes, previous midterms, homework, study guides, and the textbook.
Chapter 1: Vectors, Matrices, and Applications

Formulas and Equations

One way to parametrize a line that runs through the point $p$ in the direction of $\vec{v}$ is:

$$p + t(\vec{v}), \ t \in \mathbb{R}$$

If $\theta$ is the angle between the vectors $\vec{v}$ and $\vec{w}$ then:

$$\vec{v} \cdot \vec{w} = \|\vec{v}\|\|\vec{w}\|\cos(\theta)$$

The vector projection of $\vec{b}$ onto $\vec{a}$ is:

$$\text{proj}_{\vec{a}}(\vec{b}) = (\vec{a} \cdot \vec{b})\frac{\vec{a}}{\|\vec{a}\|^2}$$

The scalar projection of $\vec{b}$ onto $\vec{a}$ is:

$$\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2}$$

The work of a constant force $\vec{F}$ on an object with displacement vector $\vec{d}$ is:

$$W = \vec{F} \cdot \vec{d}$$

Note that if the force is not constant or the object is not moving in a straight line, then you will need to take a path integral instead.

The plane that contains the point $(x_0, y_0, z_0)$ and has normal vector $\vec{n} = (a, b, c)$ is:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

This can be simplified to $ax + by + cz = d$, where $d = ax_0 + by_0 + cz_0$.

One way to parametrize a plane that contains the point $(x_0, y_0, z_0)$ and has distinct tangent vectors $\vec{v}$ and $\vec{w}$ is:

$$(x_0, y_0, z_0) + s\vec{v} + t\vec{w}, \ s, t \in \mathbb{R}$$

Setting the above equation equal to $(x, y, z)$ and doing algebra to eliminate $s$ and $t$ will result in the plane having the form $ax + by + cz = d$. However, it is usually much quicker to just calculate $\vec{n} = (a, b, c) = \vec{v} \times \vec{w}$.

The area of the parallelogram with sides $\vec{v}$ and $\vec{w}$ is $\|\vec{v} \times \vec{w}\|$.
**Practice Problem:**
Write an equation for the plane that contains the points $(2, 0, -3)$, $(-4, -5, 2)$, and $(0, 3, -4)$ in the form $ax + by + cz = d$.

**Practice Problem:**
Find a parametric form for the line passing through the point $(1, 2)$ in the direction $(3, 4)$, which we will call $c_1(t)$. Set $c_1(t)$ equal to $(x, y)$ and eliminate $t$ to get the line into $y = mx + b$ form. Now find a different parametrization $c_2(t)$ of the same line such that $c_2(0) = (-2, -2)$ and $c_2(2) = (-5, -6)$.

**Practice Problem:**
Suppose an object is traveling due east at a speed of 5 m/s. A wind is blowing in a northwestern direction with a constant force of 2N. How much work is done by the wind over a span of 30 s.

**Practice Problem:**
Find a vector that is perpendicular to the vector $(1, 2, 3)$ with the same length. Also, find a plane perpendicular to $(1, 2, 3)$ that passes through the point $(3, 2, 1)$.

**Practice Problem:**
Write an equation for the plane that contains the point $(1, 0, 3)$ and the line $(-3, -2, -2) + t(1, 2, -1)$ in the form $ax + by + cz = d$.

**Practice Problem:**
Find the minimum distance between the point $(3, -3, -3)$ and the plane $2x + y - z = 3$.

**Practice Problem:**
Find the minimum distance between the point $(4, 2, -3)$ and the line $(1, 0, 2) + t(-1, -1, 2)$.

**Practice Problem:**
Find the vector projection of $(3, 2)$ onto $(-1, -1)$. Then find the area of the triangle with one side the vector $(3, 2)$ and another side the result of this projection.
Chapter 2: Calculus of Functions of Several Variables

Formulas and Equations

For a function \( \vec{F}(x_1, \ldots, x_m) = (F_1(x_1, \ldots, x_m), \ldots, F_n(x_1, \ldots, x_m)) \), the derivative matrix of \( \vec{F} \) is:

\[
D \vec{F} (\vec{x}) = \begin{bmatrix}
\frac{\partial F_1}{\partial x_1}(\vec{x}) & \ldots & \frac{\partial F_1}{\partial x_m}(\vec{x}) \\
\vdots & \ddots & \vdots \\
\frac{\partial F_n}{\partial x_1}(\vec{x}) & \ldots & \frac{\partial F_n}{\partial x_m}(\vec{x})
\end{bmatrix}
\]

The gradient of a function \( f : \mathbb{R}^m \rightarrow \mathbb{R} \) is:

\[
\nabla f(x, y, z) = \left( \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_m} \right)
\]

The gradient points in the direction of maximum increase and is perpendicular to level curves of the function.

The linear approximation of the function \( f(x, y) \) at \((a, b)\) is:

\[
L_{(a,b)}(x, y) = f(a, b) + \left( \frac{\partial f}{\partial x}(a, b) \right)(x - a) + \left( \frac{\partial f}{\partial y}(a, b) \right)(y - b)
\]

If you have a surface of the form \( z = f(x, y) \) then the linear approximation \( z = L_{(a,b)}(x, y) \) is a tangent plane to the surface at the point \((a, b, f(a, b))\).

When \( \vec{G} \circ \vec{F} \) is well-defined we have the following chain rule:

\[
D(\vec{G} \circ \vec{F}(\vec{x})) = D\vec{G}(\vec{F}(\vec{x}))D\vec{F}(\vec{x})
\]

Given a function \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) and a unit vector \( \vec{u} \), the directional derivative of \( f \) in the direction of \( \vec{u} \) is:

\[
D_{\vec{u}}f(a, b) = \nabla f(a, b) \cdot \vec{u}
\]
Practice Problem:
Find the domain and range of the following functions:

1. \( f(x, y) = \frac{2y^2}{x^2 + y^2} \)
2. \( f(x, y) = \frac{y}{x} \)
3. \( f(x, y) = \sqrt{xy^2} \)
4. \( f(x, y) = e^{-x^2} \)
5. \( \vec{F}(x, y) = (\ln(x), \ln(y)) \)
6. \( f(x, y) = \arctan(x/y) \)

Practice Problem:
Find \( \lim_{(x,y)\to(0,0)} \frac{3x - 6y}{\sin(x - 2y)}. \)

Practice Problem:
Find \( \lim_{(x,y)\to(0,0)} \frac{3x}{x^2 + y^2}. \)

Practice Problem:
Find \( \lim_{(x,y)\to(0,0)} \frac{\sqrt{xy}}{x^2 + y}. \)

Practice Problem:
Find \( \lim_{(x,y)\to(0,0)} (x + y)e^{-1/(x+y)}. \)

Practice Problem:
Find \( \lim_{(x,y)\to(1,1)} \frac{x - y}{x^2 - y}. \)

Practice Problem:
If the radius of a cylinder is decreasing at a rate of 2 cm/s and the height is increasing at a rate of 5 cm/s, what is the rate that the volume of the cylinder is changing when the radius is 4 cm and its height is 3 cm?

Practice Problem:
Find the tangent plane to the surface \( f(x, y) = 3x^2 - 2y^2 + 5 \) at \((1,1)\).

Practice Problem:
Find the tangent plane to the surface \( \frac{x}{z} + \frac{z}{y} = 2 \) at \((2,1,1)\).

Practice Problem:
Find the directional derivative of \( f(x, y) = 3x^2 - 2y^2 \) at the point \((2,4)\) in the direction of \((-3,4)\). What is the direction in which \( f \) is increasing most rapidly at \((2,4)\)? What is this maximum rate of increase?
Chapter 3: Vector-Valued Functions of One Variable

Formulas and Equations

The length of a continuous path $\vec{c}: [a, b] \to \mathbb{R}^2$ (or $\mathbb{R}^3$) is:

$$l(\vec{c}) = \int_a^b \|\vec{c}'(t)\| \, dt$$

A curve that has a parametrization by arc-length $\vec{c}(s)$ satisfies:

$$\int_a^b \|\vec{c}'(s)\| \, ds = b - a$$

**Practice Problem:**
Find a parametric equation for the curve that is the intersection of $x^2 + y^2 = 4$ and the surface $xz = y$.

**Practice Problem:**
Find a parametric equation $\vec{c}(t)$ for the parabola $2x + y^2 = 3$ such that $\vec{c}(0) = (1, 1)$. Then SET UP the integral that would give the length of the parabola between (1,1) and (-3,3).

**Practice Problem:**
Find a parametric equation $\vec{c}(t)$ for the ellipse $4x^2 + y^2 = 9$ such that $\vec{c}(0) = (0, 3)$ and $\vec{c}(\frac{\pi}{2}) = (-\frac{3}{2}, 0)$.

**Practice Problem:**
Find a parametrization of the curve $x^3 = 8y^2$ and use it to find the length of this curve for $x \in [0, 2]$.

**Practice Problem:**
Let $\vec{c}(t) = (e^t, e^{-t}, \sqrt{2}t)$. Find the arc length for $t \in [0, 1]$.

**Practice Problem:**
Let $\vec{c}(t) = (\cos(t), \sin(t), t)$. Reparametrize by arc length.
Chapter 4: Scalar and Vector Fields

Formulas and Equations

A point \((x_0, y_0)\) is a critical point of \(f : \mathbb{R}^2 \rightarrow \mathbb{R}\) if \(\nabla f(x_0, y_0) = 0\).

Note that your book also considers \((x_0, y_0)\) to be a critical point if at least one of \(f_x(x_0, y_0)\) or \(f_y(x_0, y_0)\) does not exist, however this type of critical point will not be a local min/max or saddle.

The discriminant \(D\) of a function \(f : \mathbb{R}^2 \rightarrow \mathbb{R}\) is:

\[
D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2
\]

The second derivative test states that if \((x_0, y_0)\) is a critical point of \(f\) then:

- If \(D(x_0, y_0) > 0\) and \(f_{xx}(x_0, y_0) < 0\), then \(f(x_0, y_0)\) is a local maximum
- If \(D(x_0, y_0) > 0\) and \(f_{xx}(x_0, y_0) > 0\), then \(f(x_0, y_0)\) is a local minimum
- If \(D(x_0, y_0) < 0\), then \(f(x_0, y_0)\) is a saddle point
- If \(D(x_0, y_0) = 0\), then the second derivative test is inconclusive

Note that in the first two cases, \(f_{xx}(x_0, y_0)\) can be replaced with \(f_{yy}(x_0, y_0)\) as \(D(x_0, y_0) > 0\) implies that both are non-negative or both non-positive.

The divergence of \(\vec{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3\) is:

\[
div \vec{F} = \nabla \cdot \vec{F} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (F_1, F_2, F_3) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}
\]

The curl of \(\vec{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3\) is:

\[
curl \vec{F} = \nabla \times \vec{F} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
F_1 & F_2 & F_3
\end{vmatrix} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) \vec{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right) \vec{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) \vec{k}
\]
**Practice Problem:**
Find the shortest distance between the surface $z = \frac{1}{xy}$ and the origin.

**Practice Problem:**
Find and classify all of the critical points of $f(x, y) = x^2y + xy^2 - 4x$.

**Practice Problem:**
Find the absolute extreme values of the function $f(x, y) = x - y$ on the disk $D = \{(x, y)|x^2 + y^2 \leq 1\}$.

**Practice Problem:**
Let $f(x, y) = xy$. Find global maximum and minimum over the region bounded by $x^2 + y^2 = 4$.

**Practice Problem:**
Let $f(x, y) = \ln(x - y + 1)$. Find global maximum and minimum over the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 1$.

**Practice Problem:**
Find the absolute minimum and absolute maximum of the function $f(x, y) = \sin(x + y)$ over the rectangle $D = \{(x, y)|0 \leq x \leq \frac{\pi}{2}, \frac{\pi}{2} \leq y \leq \pi\}$.

**Practice Problem:**
Calculate the divergence and curl of $\vec{F}(x, y, z) = (x^2, yz, e^{xz})$. 

Chapter 5: Integration Along Paths

Formulas and Equations

For a path \( \vec{c} : [a, b] \to \mathbb{R}^2 \) (or \( \mathbb{R}^3 \)) and a function \( f : \mathbb{R}^2 \to \mathbb{R} \) (or \( \mathbb{R}^3 \to \mathbb{R} \)), the path integral of \( f \) along \( \vec{c} \) is:

\[
\int_{\vec{c}} f \, ds = \int_{a}^{b} f(\vec{c}(t)) \| \vec{c}'(t) \| \, dt
\]

A path integral is independent of the parameterization of the path.

For a vector field \( F : \mathbb{R}^2 \to \mathbb{R}^2 \) (or \( \mathbb{R}^3 \to \mathbb{R}^3 \)), the scalar curl is \( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \).

A vector field \( \vec{F} : \mathbb{R}^2 \to \mathbb{R}^2 \) (or \( \mathbb{R}^3 \to \mathbb{R}^3 \)) is a gradient vector field if there exists a function \( f : \mathbb{R}^2 \to \mathbb{R} \) (or \( \mathbb{R}^3 \to \mathbb{R} \)) such that \( \vec{F} = \nabla f \).

If \( \vec{F} \) represents a force then this integral represents the work done by \( \vec{F} \) on a particle traveling along \( \vec{c} \).

If \( \vec{F} \) is a gradient vector field with \( \nabla f = \vec{F} \), then by the Fundamental Theorem of Calculus:

\[
\int_{\vec{c}} \vec{F} \cdot ds = \int_{a}^{b} \nabla f \cdot \vec{c}'(t) dt = f(\vec{c}(b)) - f(\vec{c}(a))
\]

So for gradient vector fields, this integral does not depend on the choice of path.
**Practice Problem:**
Compute the path integral of $\int_{\vec{c}} ze^{xy} ds$ where $\vec{c}$ is the line segment from $(0, 0, 0)$ to $(3, 2, 1)$.

**Practice Problem:**
Compute the path integral of $\int_{\vec{c}} f ds$ where $f(x, y) = \sqrt{x + y}$ and $\vec{c}(t)$ is the straight line from $(0, 0)$ to $(1, 1)$.

**Practice Problem:**
Compute the path integral of $\int_{\vec{c}} (x^2 + y^2 + xy) ds$ where $\vec{c}$ is the semicircle of radius 1 centered at the origin above the $y$-axis.

**Practice Problem:**
Compute the path integral of $\int_{\vec{c}} \vec{F} \cdot ds$ where $\vec{F}(x, y, z) = (y - 2z, x + z, y - 2x)$ and $\vec{c}(t) = (t, t^2 - 1, 6 - t)$, for $t \in [0, 2]$.

**Practice Problem:**
Compute the path integral of $\int_{\vec{c}} \vec{F} \cdot ds$ where $\vec{F}(x, y) = (xy, 2x - y)$ and $\vec{c}(t)$ is the line from $(0, 0)$ to $(1, 2)$.

**Practice Problem:**
Suppose that $\nabla f(x, y, z) = (y^2 z \cos(x), 2yz \sin(x), y^2 \sin(x))$. If $f(0, 1, 2) = 5$, find $f(\frac{\pi}{2}, 1, 2)$. 

A subset $D$ of $\mathbb{R}^2$ is a region of type 1 if:

$$D = \{(x, y) | a \leq x \leq b, \phi(x) \leq y \leq \psi(x)\}$$

and given a function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$\int \int_D f \, dA = \int_a^b \left( \int_{\phi(x)}^{\psi(x)} f(x, y) \, dy \right) \, dx$$

A subset $D$ of $\mathbb{R}^2$ is a region of type 2 if:

$$D = \{(x, y) | c \leq y \leq d, \phi(y) \leq x \leq \psi(y)\}$$

and given a function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$\int \int_D f \, dA = \int_c^d \left( \int_{\phi(y)}^{\psi(y)} f(x, y) \, dx \right) \, dy$$

A region that is both type 1 and type 2 is type 3.

If $f(x, y) = 1$ then $\int \int_D f \, dA$ gives the area of $D$.

If $f(x, y, z) = 1$ then $\int \int \int_W f \, dV$ gives the volume of $W$.

Given change of variables equations $x(u, v)$ and $y(u, v)$, the Jacobian of $T(u, v) = (x(u, v), y(u, v))$ is:

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Polar coordinates $(r, \theta)$ is an alternative coordinate system for $\mathbb{R}^2$ where:

$$r^2 = x^2 + y^2$$

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

The Jacobian for polar coordinates is $\left| \frac{\partial(x, y)}{\partial(r, \theta)} \right| = r$. 
Spherical coordinates \((\rho, \phi, \theta)\) is an alternative coordinate system for \(\mathbb{R}^3\) where:

\[
\begin{align*}
\rho^2 &= x^2 + y^2 + z^2 \\
x &= \rho \sin(\phi) \cos(\theta) \\
y &= \rho \sin(\phi) \sin(\theta) \\
z &= \rho \cos(\phi)
\end{align*}
\]

The Jacobian for spherical coordinates is

\[
\left| \frac{\partial (x, y, z)}{\partial (\rho, \phi, \theta)} \right| = \rho^2 \sin(\phi).
\]

Cylindrical coordinates \((r, \theta, z)\) is an alternative coordinate system for \(\mathbb{R}^3\) where:

\[
\begin{align*}
r^2 &= x^2 + y^2 \\
x &= r \cos(\theta) \\
y &= r \sin(\theta)
\end{align*}
\]

The Jacobian for cylindrical coordinates is

\[
\left| \frac{\partial (x, y, z)}{\partial (r, \theta, z)} \right| = r.
\]
Practice Problem:
Evaluate \( \int \int_{D} \cos(y^2) \, dA \) where \( D \) is the triangle with vertices \((0, 0), (0, \sqrt{\pi}), \) and \((\sqrt{\pi}, \sqrt{\pi})\).

Practice Problem:
Find the volume of the three-dimensional solid bounded by the plane \( 3x + 2y + z = 6 \) in the first octant.

Practice Problem:
Find the area of the region bounded by \( y = x^2 \) and \( y = \sqrt{x} \).

Practice Problem:
Evaluate \( \int \int_{D} xy \, dA \) where \( D \) is the region bounded by \( 1 \leq x^2 + y^2 \leq 4 \) in the first quadrant.

Practice Problem:
Evaluate \( \int \int_{W} x \, dV \) where \( W \) is the region bounded by the plane \( 3x + 2y + z = 6 \) in the first octant.

Practice Problem:
Evaluate \( \int \int_{W} z \, dV \) where \( W \) is the cone bounded by \( z^2 = -4 + x^2 + y^2 \) and \( z = 0 \).

Practice Problem:
Evaluate \( \int \int_{W} z \, dV \) where \( W \) is the three-dimensional solid that is bounded by the sphere \( x^2 + y^2 + z^2 = 4 \) and above the \( xy \)-plane.

Practice Problem:
What is the volume of the three-dimensional solid bounded by the sphere \( x^2 + y^2 + z^2 = 9 \) and the planes \( z = 1 \) and \( z = 2? \)
Chapter 7: Integration Over Surfaces, Properties, Applications

Formulas and Equations

Given a parametrization $\vec{r}(u,v)$ of a surface $S$, a normal vector $N(u,v)$ to the surface at the point $(u,v)$ is:

$$\vec{N}(u,v) = \frac{\partial\vec{r}}{\partial u}(u,v) \times \frac{\partial\vec{r}}{\partial v}(u,v)$$

Given a surface $S$ in $\mathbb{R}^3$ parametrized by $\vec{r}(u,v) : D \rightarrow \mathbb{R}^3$ and a function $f : S \rightarrow \mathbb{R}$, the surface integral of $f$ over $S$ is:

$$\int \int_S f dS = \int \int_D f(\vec{r}(u,v)) \|\vec{N}(u,v)\| dA$$

Note that this integral is independent of the parametrization of $S$. If $f(x,y,z) = 1$ then this integral gives the surface area of $S$.

If a surface $S$ can be rewritten in the form $F(x,y,z) = C$ where $C$ is a constant, then $S$ is a level curve of $F(x,y,z)$ at $C$. Thus a unit normal vector $\vec{n}$ of $S$ at $(x,y,z)$ is:

$$\vec{n}(x,y,z) = \frac{\nabla F(x,y,z)}{\|\nabla F(x,y,z)\|},$$

since $\nabla F$ is perpendicular to level curves of $F$.

If a surface $S$ in $\mathbb{R}^3$ can be written in the form $z = g(x,y)$ (i.e. you can solve for $z$) and $f : S \rightarrow \mathbb{R}$, then the surface integral of $f$ over $S$ can instead be expressed as:

$$\int \int_S f dS = \int \int_D f(x,y,z) \frac{1}{|\vec{n} \cdot \vec{k}|} dA,$$

where $\vec{k} = (0,0,1)$ and $\vec{n}$ is a unit normal vector to the surface.

Given a surface $S$ in $\mathbb{R}^3$ parametrized by $\vec{r}(u,v) : D \rightarrow \mathbb{R}^3$ and a vector field $\vec{F} : S \rightarrow \mathbb{R}^3$, the surface integral $\int \int_S \vec{F} \cdot dS$ of $\vec{F}$ over $S$ is:

$$\int \int_S \vec{F} \cdot dS = \int \int_D \vec{F}(\vec{r}(u,v)) \cdot \vec{N}(u,v) dA$$

This integral is also called the flux of $\vec{F}$ across the surface $S$. If $f(x,y,z) : S \rightarrow \mathbb{R}$ is a real-valued function, then the flux of $f$ across the surface $S$ implies finding $\int \int_S \nabla f \cdot dS$. 
Note that the sign of $\int_S \mathbf{F} \cdot d\mathbf{S}$ of $\mathbf{F}$ depends on the orientation of the parameterization $\mathbf{r}$ of $S$. So if there was another parameterization $\mathbf{r}^* : D^* \rightarrow \mathbb{R}^3$ of $S$ and $\mathbf{N}^*(u, v) = \frac{\partial \mathbf{r}^*}{\partial u} \times \frac{\partial \mathbf{r}^*}{\partial v}$ then

$$\int \int_D \mathbf{F}(\mathbf{r}(u, v)) \cdot \mathbf{N}(u, v)dA = \int \int_{D^*} \mathbf{F}(\mathbf{r}^*(u, v)) \cdot \mathbf{N}^*(u, v)dA^*$$

if $\mathbf{r}$ and $\mathbf{r}^*$ have the same orientation (i.e. $\mathbf{N}(u, v)$ and $\mathbf{N}^*(u, v)$ point in the same direction), and

$$\int \int_D \mathbf{F}(\mathbf{r}(u, v)) \cdot \mathbf{N}(u, v)dA = - \int \int_{D^*} \mathbf{F}(\mathbf{r}^*(u, v)) \cdot \mathbf{N}^*(u, v)dA^*$$

if $\mathbf{r}$ and $\mathbf{r}^*$ have opposite orientations (i.e. $\mathbf{N}(u, v)$ and $\mathbf{N}^*(u, v)$ point in opposite directions).
Practice Problem:
Compute $\int \int_S f dS$ where $f(x, y, z) = (x^2 + y^2)z$ and $S$ is the hemisphere $x^2 + y^2 + z^2 = 4$ above the $xy$-plane.

Practice Problem:
Compute the surface area of the cone $z^2 = x^2 + y^2$ and bounded above by $z = h$.

Practice Problem:
Compute $\int \int_S f dS$ where $f(x, y, z) = x^2 + y^2 + z^2$ and $S$ is the hemisphere $x^2 + y^2 + z^2 = 1$ and $x \leq 0$.

Practice Problem:
Compute $\int \int_S \vec{F} \cdot dS$ where $\vec{F}(x, y, z) = (x, y, z)$ and $S$ is the cylinder $x = y^2 + z^2$ bounded by $x = 0$ and $x = 2$ oriented with normal vector pointing outwards.

Practice Problem:
Compute the downwards flux of $\vec{F}(x, y, z) = (x, y, z)$ across the plane $x + y + z = 3$ in the first octant.

Practice Problem:
Compute the inwards flux of $\vec{F}(x, y, z) = (x, y, z)$ across the sphere $x^2 + y^2 + z^2 = 1$.

Practice Problem:
Compute $\int \int_S \vec{F} \cdot dS$ where $S$ is the paraboloid $y = 4 - x^2 - z^2$ with normal vector pointing in the positive $y$-direction and $\vec{F}(x, y, z) = (0, 3, 0)$.