AN INTRODUCTION TO TRANSFORMATIONS

Ready (Summary)	Set (Goals)
In this lesson, participants are introduced to the concept of transformations through two hands-on activities: a patty paper translation and a rubber band dilation. They learn that a transformation is a mapping of the plane onto itself, and some of the basic properties of isometries and dilations. They link the concept of congruency to isometries and the concept of similarity to dilations.	• Understand that a transformation is a function that maps the plane onto itself. • Identify some basic properties of isometries and dilations. • Link the concept of congruency to isometry. • Link the concept of similarity to dilation.

Go (Warmup)

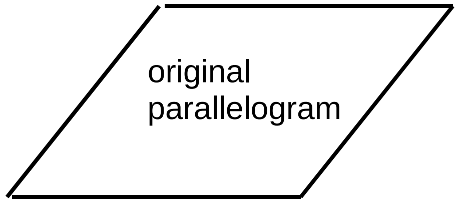
Use a piece of patty paper.

1. Trace the parallelogram below on patty paper.

2. Turn your patty paper over to the back. Using a regular or colored pencil, draw over the parallelogram on the back of the paper.

3. Put the patty paper on top of the original parallelogram. Slide the patty paper about 2 inches to the "northeast" (i.e. slide right and then up).

4. Trace over the patty paper parallelogram. Pencil markings from the back of the patty paper should transfer to this paper. Use the light markings to draw the resulting figure.



We will call this drawing technique the "two-sided transfer technique".

ABOUT TRANSFORMATIONS

A transformation of the plane is a one-to-one mapping (function) of the plane onto itself.

Refer to the warmup. You were given a parallelogram. Then, using patty paper you performed a slide to locate a new parallelogram. This action can be thought of as a transformation. The resulting figure is called the image of the original figure.

1. Color the original figure blue. Label it parallelogram *ABCD*.

2. Color the image red. Label it parallelogram *A'B'C'D'*. Be sure to mark corresponding points with the same letter.

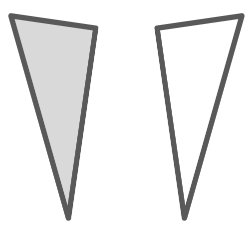
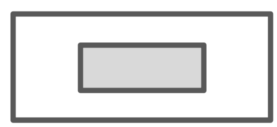
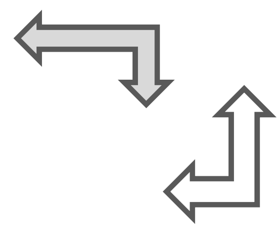
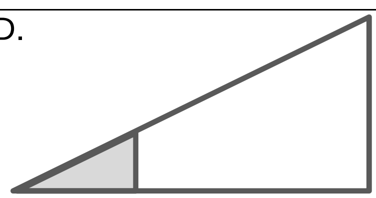
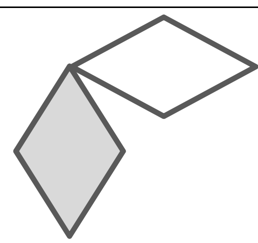
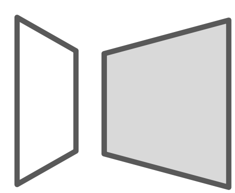
Write TRUE OR FALSE for each statement.

Under this transformation:	True or False
a. Distance is preserved. In other words, lines are taken to lines, and line segments to line segments of the same length.	
b. Parallelism is preserved. In other words, parallel lines are taken to parallel lines.	
c. Angle measure is preserved. In other words, angles are taken to angles of the same measure.	
d. Collinearity is preserved. In other words, if three points lie on the same line, then their images lie on the same line.	
e. Betweenness is preserved. In other words, if *B* is between *A* and *C* on a line, then *B'* is between *A'* and *C'* on the images.	

INTRODUCTION TO ISOMETRIES

When a transformation preserves shape and size, it is called an <u>isometry</u>. Isometry contains two Greek root words. “Iso” means equal. “Metry” means measure.

1. Here are some figures and their images under transformations. In each case, the original figure (shaded) is transformed to its image (outline only) by some transformation. Color the original figure blue and its image red. Circle each transformation that appears to be an isometry. Describe in words (1) why you believe this is an isometry, and (2) what happened to the plane under this transformation.

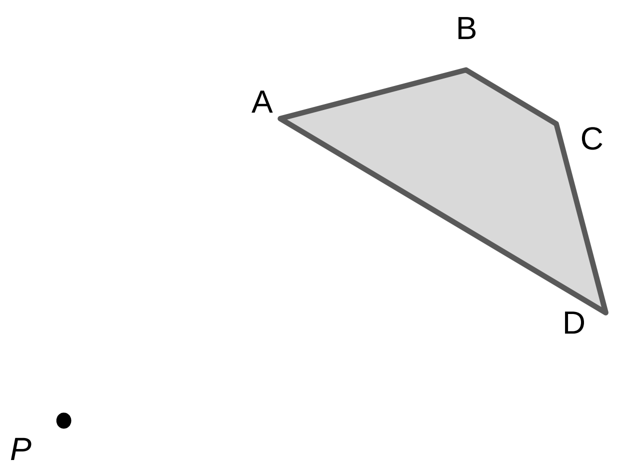
A. 	B. 	C. 
D. 	E. 	F. 

2. We say two figures are <u>congruent</u> if there is a sequence of isometries of the plane onto itself that map one figure onto the other. In other words, congruent shapes have the same shape and size. Which of the pairs of figures above appear congruent? Justify the pairs you pick.

INTRODUCTION TO DILATIONS

A **dilation** with center *P* and scale factor $k \neq 0$ is a mapping such that if point *A* is different from *P*, then the image A' lies on line *PA* and $PA' = |k| \cdot PA$

1. Use two small rubber bands. Link them together. Anchor one end of the band at point *P* with a pencil. Put another pencil into the other end of the second band. Move the pencil so that the knot linking the bands traces over the figure. You will create a new figure that is (approximately) a dilation of the first.



2. Color the original figure blue. Color the image red. Label it *A'B'C'D'*. Be sure to mark corresponding points with the same letter.

3. Draw line segments connecting point *P* to corresponding points. Find these ratios:

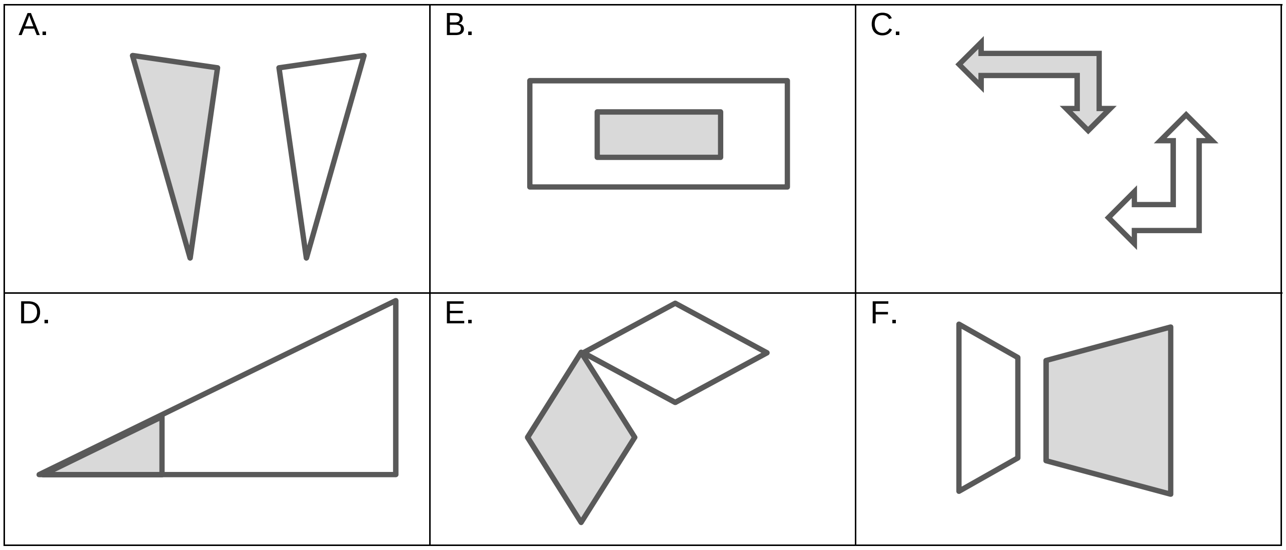
$\frac{PA'}{PA} =$ $\frac{PB'}{PB'} =$ $\frac{PC'}{PC} =$ $\frac{PD'}{PD} =$

4. What do you notice about these ratios?______________________________

5. What is the scale factor for this dilation?___________

PROPERTIES OF ISOMETRIES AND DILATIONS

1. Recall the transformations from the previous page. Which pairs of figures appear congruent?



2. We say two figures are <u>similar</u> if there is a sequence of isometries and dilations of the plane onto itself that maps one figure onto the other. In other words, similar figures have the same shape, but may be a different size. Which of the figures above appear to be similar?

3. Write TRUE OR FALSE for each statement.

Under this transformation:	A	B	C	D	E	F
a. <u>Distance</u> is preserved.						
b. <u>Parallelism</u> is preserved.						
c. <u>Angle measure</u> is preserved.						
d. <u>Collinearity</u> is preserved.						
e. <u>Betweenness</u> is preserved.						

6. Which properties appear to hold for all isometries (congruent figures)? _______________

7. Which properties appear to hold for all dilations (similar figures)? __________________