#8.1.3. Suppose that \( D \) is the region \([-1,1] \times [-1,1] \), \( P(x,y) = -y \), and \( Q(x,y) = x \). Explicitly calculate \( \int_{\partial D^+} P \, dx + Q \, dy \) and \( \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dxdy \) individually. Verify that Green’s theorem holds in this situation: i.e. that these two integrals are equal.

#8.1.4. Repeat #8.1.3 with \( D = [-1,1] \times [-1,1] \), \( P(x,y) = x \), \( Q(x,y) = y \).

#8.1.11(a). Repeat #8.1.3 with \( D = \{(x,y) : x^2 + y^2 \leq R^2 \} \), \( P(x,y) = xy \), \( Q(x,y) = -yx^2 \).

#8.1.11(c). Repeat #8.1.3 with \( D = \{(x,y) : x^2 + y^2 \leq R^2 \} \), \( P(x,y) = xy \), \( Q(x,y) = xy \).

#8.1.14(b). Suppose that \( D, P, Q \) satisfy the hypotheses of Green’s theorem. Prove the following equality:

\[
\int_{\partial D^+} \left( Q \frac{\partial P}{\partial x} - P \frac{\partial Q}{\partial x} \right) \, dx + \left( P \frac{\partial Q}{\partial y} - Q \frac{\partial P}{\partial y} \right) \, dy = 2 \iint_D \left( \frac{\partial^2 Q}{\partial x \partial y} - \frac{\partial^2 P}{\partial x \partial y} \right) \, dxdy.
\]

#8.1.17. For any pair of arbitrary real numbers \( 0 < a < b \), repeat #8.1.3 with \( D = \{(x,y) : a \leq x^2 + y^2 \leq b \} \), \( P(x,y) = 2x^3 - y^3 \), \( Q(x,y) = x^3 + y^3 \). Note that \( D \)’s boundary consists of two distinct circles, with the inner circle oriented in the clockwise direction and the outer oriented in the counterclockwise direction.

#8.1.19(a). Verify the divergence theorem for \( F = xi + yj \) and \( D = \{(x,y) : x^2 + y^2 \leq 1 \} \). In other words, show that \( \int_{\partial D} F \cdot n \, ds \) is equal to \( \iint_D \text{div} \, F \, dA \).

#8.1.20. Let \( D \) denote the unit disk, \( P(x,y) = -\frac{y}{x^2 + y^2} \), and \( Q(x,y) = \frac{x}{x^2 + y^2} \). Show that the claim in Green’s theorem does not hold here: i.e. \( \int_{\partial D^+} P \, dx + Q \, dy \) is not equal to \( \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dxdy \). Explain why.