| Math 108a | Professor: Padraic Bartlett |
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# Homework 2: Vector Spaces

*Due Thursday, Oct. 10, 3pm, South Hall 6516* — *UCSB 2013*

Remember: homework problems need to show work in order to receive full credit. Simply stating an answer is only half of the problem in mathematics; you also need to include an argument that persuades your audience that your answer is correct! As always, if you have any questions, feel free to contact either Shahab or I via email or office hours. Have fun!

1. Let $\mathcal{C}(\mathbb{R})$ denote the collection of all continuous functions on the real numbers. Is this a vector space? Prove your answer. (Feel free to simply state that associativity, commutativity and distributivity are inherited from the real numbers: I'm more interested in the other axioms.)

2. Let $\not\mathcal{C}(\mathbb{R}) \cup \{0\}$ denote the collection of all discontinuous functions on the real numbers, along with the identically-0 function $f(x) = 0$. Is this a vector space? Prove your claim.

3. Consider the vector space $\mathbb{R}[x]$, the collection of all polynomials with real-valued coefficients. Consider the following pair of subsets of $\mathbb{R}[x]$:

$$S = \{p(4) = 0 \mid p(x) \in \mathbb{R}[x]\},$$
$$T = \{p(1) = 5 \mid p(x) \in \mathbb{R}[x]\}.$$

   In other words, $S$ is the collection of all polynomials that are equal to 0 at $x = 4$, and $T$ is the collection of all polynomials that are equal to 5 at $x = 1$.

   Is $S$ a subspace of $\mathbb{R}[x]$? How about $T$? Prove your claims.

4. In class, we proved that the collection of all polynomials of degree 5 or less was a subspace, and therefore was a vector space. Call this space $\mathcal{P}_5(\mathbb{R})$. Can you find a set of six polynomials $S = \{p_1(x), p_2(x), p_3(x), p_4(x), p_5(x), p_6(x)\}$ such that none of these polynomials are degree 3, such that the span of $S$ is $\mathcal{P}_5(\mathbb{R})$?

5. Let $V$ be a vector space, and $\{\vec{v_1}, \ldots \vec{v_n}\}$ a set of vectors that spans $V$. Show that the set

$$\{\vec{v_1} - \vec{v_2}, \vec{v_2} - \vec{v_3}, \ldots, \vec{v_{n-1}} - \vec{v_n}, \vec{v_n}\}$$

   also spans $V$.

6. Suppose that the set $\{v_1, \ldots v_n\}$ is linearly independent. Is the set

$$S = \{\vec{v_1} - \vec{v_2}, \vec{v_2} - \vec{v_3}, \ldots, \vec{v_{n-1}} - \vec{v_n}, \vec{v_n}\}$$

   linearly independent? How about the set

$$T = \{\vec{v_1} - \vec{v_2}, \vec{v_2} - \vec{v_3}, \ldots, \vec{v_{n-1}} - \vec{v_n}, \vec{v_n} - \vec{v_1}\}?$$

7. How long did you spend on this set? (This question is just for calibration purposes, and will not change your score or be in any way attached to your name.)