Math 108a Professor: Padraic Bartlett

Homework 3: Properties of Vector Spaces

Due Thursday, Oct. 10, 3pm, South Hall 6516 *UCSB 2013*

Remember: homework problems need to show work in order to receive full credit. Simply stating an answer is only half of the problem in mathematics; you also need to include an argument that persuades your audience that your answer is correct! As always, if you have any questions, feel free to contact either Shahab or I via email or office hours. Have fun!

1. Find four vectors in $\mathbb{R}^3$, such that the dot product of any two of them is negative.

2. In class, we asked the following question: for what values of n can you find a basis for $\mathbb{R}^n$ with the two properties $\star, \ddagger$ described below?

   $\star$. Every vector in the basis is made up out of entries from ± 1.

   $\ddagger$. The dot product of any two vectors in the basis is 0.

   We found examples of such bases for $\mathbb{R}^1, \mathbb{R}^2$ and $\mathbb{R}^4$, and showed that no such basis exists for $\mathbb{R}^3$.

   Find a basis for $\mathbb{R}^8$ with the two properties $\star, \ddagger$.

3. Show that there is no basis for $\mathbb{R}^7$ with the two properties $\star, \ddagger$.

4. Let S be the collection of all polynomials with degree at most 2 that has a root at $x = 7$: in other words:

   $$S = \{p(x) = a + bx + cx^2 \mid p(7) = 0\}.$$

   Explain, briefly, why this is a vector space. (Feel free to reference the answer and logic used in HW#2, problem 3.)

   After you do this, find a basis for S.

5. Consider the vector space $\mathbb{R}[x]$, consisting of all polynomials with finite degree with real-valued coefficients. Find a basis for this space. Does this space have a basis with finitely many elements?

6. Consider the following map $L : \mathbb{R}^4 \to \mathbb{R}^4$:

   $$L((a, b, c, d)) = (d, c, b, a)$$

   Is this a linear transformation?

7. How long did this set take you? (As always, asked for calibration purposes.)