

Homework 9: Latin Squares

*Due 12/3/13, at the start of class**UCSB 2013*

Instructions: Do problems here until you have spent about 90 minutes working seriously on these questions. Have fun!

Homework Problems

1. Is the following partial Latin square P always completable to a proper Latin square? (Assume that P is of order ≥ 3 .)

$$\begin{bmatrix} 1 & - & - & \dots & - \\ - & 2 & - & \dots & - \\ - & - & 3 & \dots & - \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ - & - & - & \dots & n \end{bmatrix}$$

2. What is the smallest possible number of filled cells in a 8×8 partial Latin square P , so that there is exactly one way to complete P to a proper Latin square?
3. Show that if P is a 4×4 partial Latin square in which at most 3 cells are filled, P can be completed to a proper Latin square.
4. Show that if P is a $n \times n$ partial Latin square containing $\leq n - 1$ filled cells, then P can be completed to a proper Latin square.
5. Call a $n \times n$ partial Latin square $\frac{1}{4}$ -dense if no row, column, or symbol is used more than $\frac{n}{4}$ many times. (In other words, we want to have no more than $n/4$ filled cells in any row or column, and also don't want any symbol to be used more than $n/4$ of the time.) Can any $\frac{1}{4}$ -dense partial Latin square be completable to a proper Latin square?
6. Generalize this: for any n , find a $n \times n$ partial Latin square P such that no rows, columns, or symbols are used more than $\frac{n}{4} + 1$ many times, such that P cannot be completed to a proper Latin square.
7. Show that if P is a $n \times n$ partial Latin square, n even, where the upper-quadrant $\frac{n}{2} \times \frac{n}{2}$ is filled and the rest is blank, then P can be completed to a Latin square.