Math/CS 103 Professor: Padraic Bartlett

Homework 15: Elementary Matrices

Due 11/18/13, at the start of class. *UCSB 2013*

There are a number of problems on this set. Problems from the first two sections are worth half a point apiece; problems from the third section are worth a point apiece. Do **three** points worth of problems. Have fun!

1 Desert Islands and Elementary Matrices

The scenario for this section is the following: suppose that you're stranded on a desert island, with nothing but a large box containing all of the 3×3 elementary matrices. What matrices can you create?

At the end of class, we noticed that we can make things like

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}.$$

What else can we create? Specifically: consider the following six matrices. Come up with a sequence of elementary matrices that we can compose together to create that matrix.

1. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 5 & 8 \\ 13 & 21 & 34 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
4. $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$
5. $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$
6. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

2 Desert Islands: Now With Less Matrices

Suppose you're in the same situation as before, but parrots on your desert island have stolen all of your elementary matrices of the form $E_{\text{multiply entry k by 0}}$. So you still have all of your "swap two things" elementary matrices, and all of your "add copies of one row to another" matrices, but you only have the "multiply a row by λ" matrices when $\lambda \neq 0$.

Consider the following four matrices that we made in section 1. Show that we can no longer make these matrices.

1. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 5 & 8 \\ 13 & 21 & 34 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

3 Elementary Matrices: Now With Less Desert Islands

1. Take **any** 3×3 matrix A. Show that we can create A with an appropriate combination of elementary matrices.

2. Suppose that A is a 3×3 matrix that we cannot invert. Prove that we cannot write A A as a composition of elementary matrices, if we do not get to use the elementary matrices of the form $E_{\text{multiply entry k by 0}}$.