Choose **three** of the **five** problems below to complete by next class. Be ready and able to present your solutions if you have them, or your questions if you don't solve the problems!

1. The deep-sea navigation business from HW#1 has decided to start offering tours of the surrounding ocean! A **tour**, for our purposes, is a set of instructions that we can give a submarine (i.e. a sequence of engines that we start and stop) such that after performing all of these instructions, the submarine has returned to where it started.

   For example, consider a submarine with three engines, which (when ran for a hour from any given location) can move it either $(0, -2, -1), (0, 1, -2)$, or $(0, 3, 0)$ units from where the submarine started. A valid tour is illustrated below:
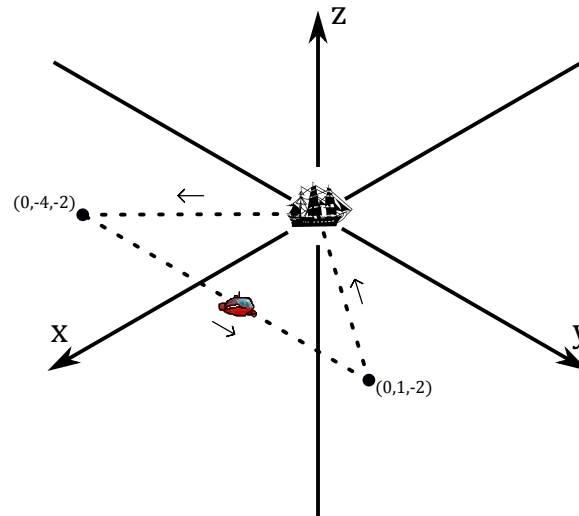
   We have three submarines available to potentially offer tours in:

   : Submarine 1 has two turbines attached to it, which from any given location can move it either $(1, -2, -1)$ or $(2, -4, -2)$ units.

   : Submarine 2 has three turbines attached to it, which (when ran for a cycle from any given location) can move it either $(1, 2, -1)$, $(-2, -4, 2)$, or $(1, 1, -3)$ units.

   : Submarine 3 has three turbines attached to it, which from any given location can move it either $(-1, -1, 0)$, $(0, -1, -1)$ or $(-1, 0, -1)$ units in any given direction.

   Like in HW#1, these submarines can fire their engines for fractional amounts of cycles and run their engines in "reverse." However, they have the following restriction: once an engine is used once in any tour, it cannot be used again (due to danger of overheating.) (This condition applies to every problem on this set, by the way.)

   Which of our three submarines can perform a tour? Why or why not?

2. Create a submarine with two engines that cannot perform a tour. Classify all such submarines: i.e. describe a simple rule that you could apply to any two-engine submarine design to determine if it can perform a tour.

3. (a) Can you create a submarine with three engines that can perform a tour, but such that removing any one engine makes performing a tour impossible? Either construct an example, or explain why no such example exists.

   (b) Similarly: can you create a submarine with four engines that can perform a tour, but such that removing any one engine makes performing a tour impossible?

   (c) Last variation: can you create a submarine with five engines that can perform a tour, but such that removing any one engine makes performing a tour impossible?

4. Oh no! You've just discovered that the submarine you're on has been sabotaged!

   Specifically: your submarine normally has three engines, which when ran for a cycle move our submarine either $\vec{A}, \vec{B}$, or $\vec{C}$ units. Using these three engines, you normally could travel to any arbitrary point in the ocean. However, someone has snuck onto your ship and fused some of your engine controls together.

   Now, whenever you go to use the first engine $\vec{A}$, it fires the second one $\vec{B}$ in reverse, resulting in a motion of $\vec{A} - \vec{B}$. Similarly, whenever the second engine $\vec{B}$ is fired the third engine $\vec{C}$ is fired in reverse, resulting in the motion $\vec{B} - \vec{C}$. Finally, whenever the third engine $\vec{C}$ is fired the first engine $\vec{A}$ runs in reverse, resulting in a motion of $\vec{C} - \vec{A}$.

   This sabotage has just taken place out in the deep sea; you need to get back to base to repair your controls. Regardless of where you are in the sea, can you always make it back to base? Or is it possible that you're stranded? If you can always get back, describe how; if you're stranded, give an example that shows how.

5. You're still aboard the sabotaged submarine from problem 4, However, using your amazing 1337 hacker skillz, you've managed to fix the third engine: now when you fire the third engine, you just get $\vec{C}$. Can you get back to base now?