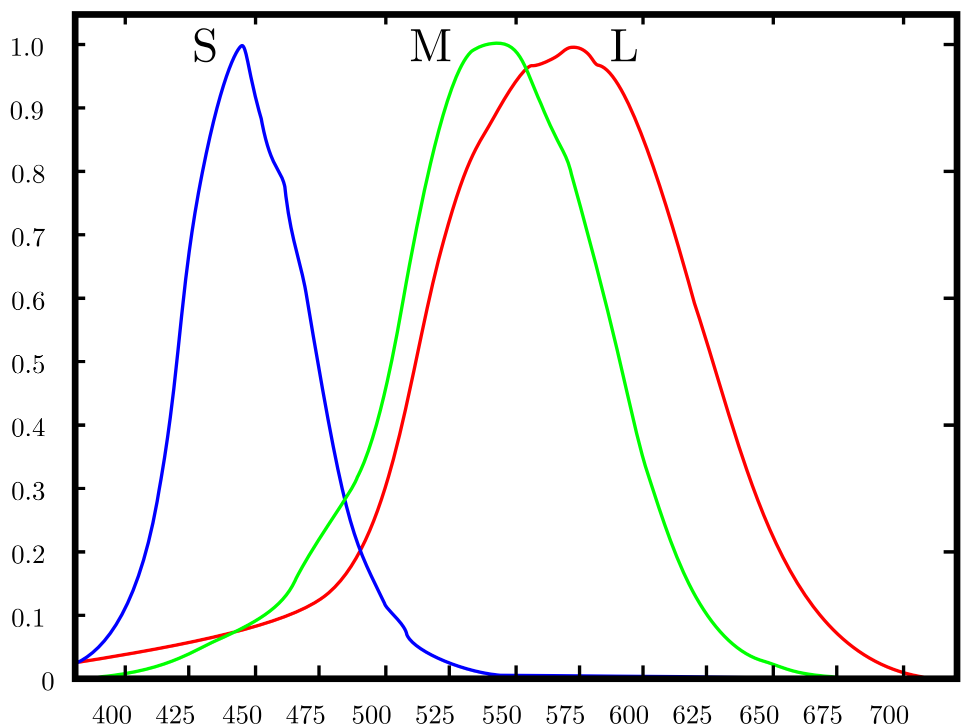
Math/CS 103	Professor: Padraic Bartlett

Homework 8: Color Space

Due 10/21/13, at the start of class — *UCSB 2013*

Hi! A brief physics and biology lesson.

- Light! It exists. In particular, a given photon of light has some set wavelength, measured in this problem set in nanometers (nm).
- You have eyes! You use them to detect light.
- Specifically, your eyes detect color via the use of various types of cells called **cones**, that live in your retina (i.e. the back of your eye) and are activated when light hits them.
- There are three types of cones: L-cones, M-cones and S-cones. When these cones are struck by a given beam of light, they send a signal to your brain indicating that they have been struck. The strength of these signals varies from person to person, but typically matches the three normalized graphs below:

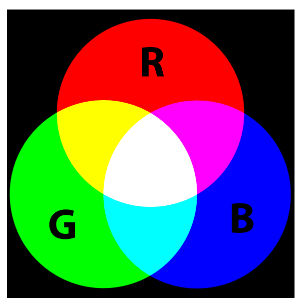


So, for example, if a beam of light with wavelength 500 nm were to be shined at someone, their L-cones would likely send a signal of strength about 0.28, their M-cones would send a signal of strength about 0.45, and their S-cones would send a signal of strength about 0.1. We can think of this signal as a triple $(L, M, S) = (0.28, 0.45, 0.1)$.

This problem set contains five questions dealing with how we perceive and work with color. Choose **three** of the **five** problems below to complete by next class. Be ready and able to present your solutions if you have them, or your questions if you don't solve the problems!

1. (a) The color red is associated to light photons with wavelengths between $635 - 700$ nm; similarly, the color green is associated to light photons with wavelengths between $490 - 560$ nm, and the color blue is associated with light photons with wavelengths between $450 - 490$ nm. For each of these three spectrums of wavelengths of light, choose an appropriate value somewhere in the middle of this spectrum. For these chosen values, use the charts on page 1 to come up with appropriate triples (L, M, S), that describe the signals your eyes send when hit with this light.

   (b) The color yellow is associated to light photons with wavelengths between $560 - 590$ nm. Pick a value from the middle of this spectrum and find a triple (L, M, S) that corresponds to this wavelength. If you've ever played around with lights or prisms at a younger age, you might remember that **combining equal amounts of red and green light** generates something that looks like yellow. Does this hold true? In other words: is the (L, M, S) triple you got for yellow light roughly the average of the (L, M, S) triples you got for red and green?



   (c) Can you find another color/wavelength that can be "simulated" by combining your red/green/blue (L, M, S) triples?

   (d) In a sense, we can regard the triples (L, M, S) as all of the possible "colors" your brain can experience. Are there any colors your brain can perceive, that cannot be generated by light? Justify your claim.

2. Another way in which color is frequently studied is not with respect to perception, but rather with respect to **reproducibility**. Specifically: TV screens, LCD monitors, and most any modern display system works via the following method:

   - LED's (light-emitting diodes) are small semiconductors that emit light of a particular wavelength when current is passed through them. It can be tricky to make efficient LED's to output a specific wavelength of light, but we are relatively good as a society at creating LED's that output light in the red ($620 - 645$ nm), green ($520 - 550$ nm) and blue ($460 - 490$ nm) wavelengths.

- Monitors, loosely speaking, can be thought of as a very tightly-packed grid of triples of LED's, with each triple containing one green LED, one red LED, and one blue LED. These triples can be thought of as "pixels," with respect to the images that we want to display on a given monitor. The idea here is that with these three pixels, we can generate almost all of the colors that visible light can generate (much like how we "simulated" yellow by combining red and green in problem 1.)
- Whenever your computer activates a pixel, it does so by assigning the three red/green/blue LED's in that pixel three intensities (scaled so that they range from 0 to 1). I.e. $(0.2, 0.2, 0.2)$ would generate a pixel that fires the red, green and blue LED's all at .2 intensity. $(0.4, 0.1, 0)$ would fire the red LED at 0.4 intensity, the green LED at 0.1 intensity, and not fire the blue LED.

(a) Construct a map T that takes a pixel (R, G, B) and determines its corresponding (L, M, S) value that a human eye would perceive.

(b) The color white, and various shades of gray all the way down to black, are seen whenever light sources excite the L,M and S cones equally. What triples (R, G, B) come close to corresponding to white/ these shades of gray?

3. Consider the map you came up with in part 2. Is it a linear map? Do all of its inputs correspond to (L, M, S) triples that your brain can actually experience?

4. Protanopia is a condition affecting about 1% of men and 0.1% of women, where an individual is missing all of their L-cones. This is often called "red-green colorblindness," as individuals suffering from protanopia cannot distinguish the colors red and green.

   (a) Explain why people missing their L-cones cannot tell red and green apart.

   (b) Create a linear map T that takes in (R, G, B) triples and outputs (R, G, B) triples, such that the resulting image simulates what people with protanopia suffer from.

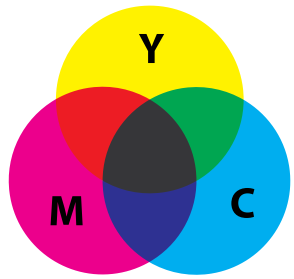
   (c) Suppose you have an image whose colors are primarily yellow, green and blue. Create a map that makes this image viewable and understandable by someone suffering from protanopia.

5. When a printer prints an image, it does so with a set of inks called $CMYK$:

   - C, denoting the color cyan, which is visualized as a sort of blue. You can create cyan-colored pixels by mixing equal parts green and blue in your (R, G, B) triples.
   - M, denoting the color magenta, which is a sort of purple. You can create magenta-colored pixels by combining equal parts red and blue in your (R, G, B) triples.
   - Y, denoting the color yellow. You can create yellow-colored pixels by combining equal parts red and green in your (R, G, B) triples.
   - K, denoting the key tone, which is black.

These pigments combine in the way you remember from elementary school and playing with paints: i.e.

- Cyan and yellow combine to make green;
- Cyan and magenta combine to make red; and
- Yellow and magenta combine to make blue.



In theory, the tone K is superfluous, because combining all three colors should yield black. In practice, modern inks don't really do this and instead just become some sort of muddled brown.

(a) Can you make a map T that sends a triple (R, G, B) to a quadruple (C, M, Y, K), so that the pixel (R, G, B) looks the same as the combination of inks given by combining (C, M, Y, K) in the proportions given?

(b) Can any such map be linear?