Math/CS 103 Professor: Padraic Bartlett

Syllabus for Math/CS 103

Weeks 1-10 *UCSB 2013*

Basic Course Information

- Professor: Padraic Bartlett.
- Email: padraic@math.ucsb.edu.
- Class time/location: MWF 11:30-12:50, Building 494, Room 160B.
- Office hours/location: TTh 1-2pm, South Hall 6516. Additionally, I am teaching a different linear algebra class (Math 108a), whose office hours are on TTh 2-3pm; you are welcome to attend these office hours instead if they work better for you. Finally, if neither of these times work well for you, I can meet students outside of these times by appointment; email me and we'll set something up!
- Course webpage: on GauchoSpace. If GauchoSpace is down, or you are otherwise having difficulty getting access, a second copy of the website can be found here: http://math.ucsb.edu/~padraic/mathcs103_2013/mathcs103_2013.html
- TA: TBA.
- TA office hours/location: TBA.

Course Description

This course is intended to change your life.

This might be surprising, given that 103A's blurb in the course catalog describes it as a course on "systems of linear equations, matrix algebra, determinants, vector spaces and subspaces, basis and dimension, linear transformations, eigenvalues and eigenvectors, diagonalization, and orthogonality." Don't get me wrong; eigenvectors are pretty cool, but "life-changing" isn't the phrase most people would use to describe them. So what do we mean by this?

Consider most of the math classes you've taken before. You go to lecture, copy down notes, solve stacks of integrals and box your answers, get a 92%, go onto the next topic. Repeat for the entirety of high school and Calc 1-3.

Contrast this with what mathematics is like outside of school. In industry, no-one will ask you to calculate a stack of derivatives or solve some pile of equations: that's what Mathematica is for. Instead, you're going to see open-ended problems. You'll be advising supply firms on how much product to buy; you're going to figure out how much weight a bridge can support without failing; you'll be determining how many flights your company needs to schedule between LAX and PEK next year.

Furthermore, when you get your answers, you won't be able to just say to your project team "Oh, 38 tons" and walk away; you're going to have to explain to them **how** you derived your answer, so they can understand it and double-check it. Furthermore, if you get something wrong, it's not like your bridge gets "$83\%, B-$" stamped on the side: you have to go back and **fix your answers** until they are **right**.

If you go into academia, this all counts double. When you're doing research in mathematics, you're often not just discovering the answers but also the questions; more of your time is spent on creating definitions and concepts than actually calculating anything. To illustrate this point, here is a page from Robertson et. al.'s paper that proves the four-color theorem[1]. Notice how this is almost entirely words and pictures!

THE FOUR-COLOUR THEOREM 15

a tri-colouring modulo $\phi(X)$, κ say. The restriction of κ to $E(H)$ is a tri-colouring of H, since $\phi(X) \cap E(H) = \emptyset$; and so its lift, λ say, via ψ belongs to $\mathscr{C}_1$ and hence to $\mathscr{C}_3$. But for $e \in E(S)$, let $\kappa'(e) = \kappa(\phi(e))$; then κ' is a tri-colouring of S modulo X, and λ is its restriction to R. This contradicts that X is a contract for S, and the result follows. ∎

4. UNAVOIDABILITY

In this section we prove (2.3). A *cartwheel* is a configuration W such that there is a vertex w and two circuits C_1, C_2 of $G(W)$ with the following properties:

(i) $\{w\}, V(C_1), V(C_2)$ are pairwise disjoint and have union $V(G(W))$

(ii) C_1 and C_2 are both induced subgraphs of $G(W)$, and $U(C_2)$ bounds the infinite region of $G(W)$

(iii) w is adjacent to all vertices of C_1 and to no vertices of C_2.

It follows that the edges of $G(W)$ are of four kinds: edges of C_1, edges of C_2, edges between w and $V(C_1)$, and edges between $V(C_1)$ and $V(C_2)$. We call w the *hub* of the cartwheel. See Fig. 3.

To avoid confusion, let us stress that Fig. 3 is a picture of W, not of the free completion of W; the free completion would have *three* concentric circuits around w.

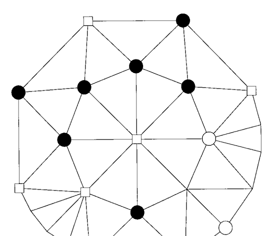

FIG. 3. A cartwheel.

If you're in this classroom, there's a pretty good chance you're interested in being a mathematician. That page above is what math in the real world looks like. Therefore, this is what math is going to look like in our classroom. This means we're going to do a number of things in this classroom that you may not have done before in a math class:

[1] The four-color theorem is a famous result in graph theory about the following problem: take any map, and suppose you want to give each country on this map its own color so that no two countries that touch have the same color. How many colors do you need? Perhaps surprisingly, if you assume that every country is **contiguous** (i.e. connected, so we don't allow things like Alaska and the US, or Kaliningrad and Russia), it turns out you only need four colors! This problem was first posed in 1852, and was only solved in 1976; its proof is notorious because a large chunk of it relies heavily on computer-aided search. No completely human-written proof is known.

1. Lectures will be very interactive; I will ask questions every two or three minutes in class, and expect you to contribute examples, ask questions, and think on the spot throughout the class.

2. At the end of every lecture, I will give out a few (around three) homework problems based on what we discussed as a group in class. These will be due at the start of the next class. Homework problems will typically be open-ended, involve multiple stages, and involve more thought than "just apply theorem x."

3. Furthermore, **the first half of each class** will be set aside for students to present their solutions to these problems to the rest of the class at the board. Again, this is what mathematics is like in the real world. It is not enough to simply know the answer; you have to be able to explain to others why you are **correct.**

4. Homework problems will be graded with respect to two criteria:

   (a) Did you arrive at the correct answer, using a sound chain of logical statements?

   (b) Does your work cleanly and carefully lay out a full and complete solution to the problem at hand? I.e. could your solution be used as an example in a linear algebra textbook?

   For each of these criteria, problems will either receive a check (if they satisfy the criteria) or no check (if they do not.) Partial credit will not be given, and the TA and I will be pretty strict in our interpretations of "full and complete solution." (A heuristic I tend to use is the following: would any step take an average student in this course no more than ten seconds to figure out? If so, then that step should be written down.)

5. Homework will be returned by the next class period. Up to a week after returning said homework, students may revise their solutions to these graded problems, and resubmit their work: these problems will be regraded, and the new scores will replace your old scores. This policy is part of why we feel comfortable being pretty strict with the grading process above: revising your work is something I want you to get in the habit of.

Course Evaluation

There are four components of your grade in Math/CS 103A:

1. **Homework** (25%.) As noted before, there will be daily problem sets: problems will be handed out at the end of one class and due at the start of the next. After they are graded, sets can be revised and turned in within a week for a regrade.

2. **Participation** (25%). The first half of every class will consist of student presentations of the homework problems that were due that day. In order to receive a full participation grade, you need to regularly present correctly-solved problems in class to your peers. Specifically: suppose that Y students are enrolled in our course, and

that that X homework problems are assigned throughout the course. If every student were to participate equally, you would assume that each student would present the answers to roughly $\frac{X}{Y}$ problems. However, this isn't entirely realistic: some students will be more introverted than others, some students will get problems wrong, and some students will be sick or miss certain days. Therefore, a more reasonable expectation for what students should do to get full credit is more like $\frac{2}{3} \cdot \frac{X}{Y}$. As of the start of September, when I wrote this paragraph, our class looks like it will contain about 13 students, and have about 25 problem sets each containing about three questions. If this remains true, then any student presenting four or more correct problems over the course — i.e. an average of one presentation every two weeks, if most of your presented problems are correct — will receive full credit here. Students who participate more will receive extra credit and students who participate less will have their score scaled down, as appropriate.

To insure that class isn't dominated by any one student and that all students have the opportunity to present problems, we will use the following system for deciding who presents a problem:

(a) Before class, I will post a list of all of the homework problems assigned that week.

(b) Students will go and sign up for problems they feel they can present to their peers.

(c) I will take the list, and for each problem select the student who has the **least** number of presentations to their name to go up and present their solution. If there is a tie, an appropriately random tiebreaking process will be chosen and used.

(d) If there is a student with a materially different proof, we may have multiple proofs presented in class; this won't always be a thing we have time for, but when we can I will try to make this possible.

3. **Exams** (50%.) There will be a midterm on November 1, and a final (location TBA) on December 12th, from 12-3pm. (This may change.) Each will have a take-home and in-class portion, and each will be worth 25% of your final grade. No make-up tests will be given, except under truly extraordinary circumstances. If you have a serious conflict with either of these dates, please let me know as soon as possible.

This course is pass-fail. Any student above a 70% will definitely receive full units for the course. It is likely that this bar will be adjusted downwards, depending on how students do on the homework, midterm, and final. I will keep the class updated periodically throughout the course with averages, so students know where they are at.

Collaboration/resources policy

Collaboration is allowed (and indeed encouraged) on the homework sets; mathematics at the research level is a collaborative activity, and there is no reason that it should not also be this way in a classroom. Work with your classmates!

Similarly, mathematics **is** a research activity; I would claim that banning resources like textbooks, Wikipedia, Mathematica, etc. is something of a fool's errand, and contradictory to the spirit in which we pursue research as researchers ourselves.

The only things that we ask of you are the following:

- Write up your work separately, and only write up solutions you understand fully.
- When writing up your own work, you cannot simply cite any paper: you have to write up the proofs of any results you're planning to use, and do so in your own words. The only exception to this policy is for results in the online lecture notes or from previous HW sets.
- Don't post questions to online messageboard-style services.

The tests will have their own resource and collaboration policies, which will be printed on the test.

If you have any questions on the collaboration policy, please email me and I'll be glad to clarify matters.

Course Textbook

There is no primary textbook for this course; this is because most linear algebra textbooks either (a) only focus on calculations or (b) assume you already know all of linear algebra. Consequently, what I'm going to do throughout the course is type up all of my lecture notes[2] and post them on the course website. Between those and your homework sets, you should be able to do anything.

That said, books are nice. One good text is Lay's Linear Algebra and its Applications, which will cover almost every topic we hit along with lots of examples. It is more calculation- and matrix-focused than our class, which is more geometric in nature; but it's not a bad book. Friedberg and Spence's Linear Algebra is also great. Treil's humorously-named Linear Algebra Done Wrong is another solid book, that has the unique advantage of being free (either follow the link above or just search for the name of the text to find an electronic copy.)

[2]To type up my lecture notes, I use a program called LATeX. If you're going to be a math major, you may want to consider learning how to use LATeX yourself, as it is the tool that mathematicians use to type up pretty much anything. If you would like to learn how to use this, feel free to contact me or stop by office hours; I'd be glad to explain the basics!

Course Goals and Timeline

The following is a very rough timeline for the topics discussed each day in the course. We will likely deviate from this plan as the course progresses, depending on how quickly people pick up certain topics and where student interests lie; so don't take this outline too seriously! I would expect us to end the course within about six classes of this plan. Problem sets will typically draw on the material either just discussed in lecture or that we will discuss in the next lecture, as appropriate.

9/27: Intro; vector space operations.

9/30: Span, linear independence/dependence.

10/2: Dimension and basis.

10/4: Other vector spaces, 1/2.

10/7: Other vector spaces, 2/2.

10/9: Dot products and their meanings.

10/11: Review!

10/14: Defining linear transformations.

10/16: Domain, null space, range.

10/18: Linear transformations and matrices.

10/21: Linear maps between distinct spaces.

10/23: A closer look at null space and range.

10/25: Composing matrices.

10/28: Incidence matrices.

10/30: Review!

11/1: **Midterm.**

11/4: Systems of linear equations.

11/6: Matrices and linear equations.

11/8: Elementary matrices.

11/11: **Veteran's Day**; no class.

11/13: Understanding the inverse of a matrix.

11/15: Review!

11/18: The determinant: definition.

11/20: The determinant: interpretation.

11/22: The determinant: uses.

11/25: An introduction to eigenstuff.

11/27: The characteristic polynomial.

11/29: **Thanksgiving**; no class.

12/2: Diagonalization.

12/4: Winter preview: LA and diff. eqns.

12/6: Final review session!

12/9: **Final,** 8-11am.