Math/CCS 103	Professor: Padraic Bartlett

Homework 10: Presentations (Eric B. and Stephen)

Due Friday, week 5 *UCSB 2014*

Do **two** of the **five** problems below!

1. (Stephen) In my presentation, we discussed a puzzle that involved permutations of the list of numbers $(1, 2, ..., n)$. We describe it again here:
   - You are given an arbitrary permutation of the numbers $\{1, 2, ..., n\}$ in the form $(t_1, t_2, ..., t_n)$.
   - The "solved" permutation is $(1, 2, ..., n)$.
   - Your goal is to go from the scrambled permution to the solved permutation. The catch is that you are only allowed to sort the numbers by cyclically permutating consecutive triples of numbers: i.e. $...abc... \to ...bca... \to ...cab....$ Also, $n \geq 3$ (duh).

   (a) Come up with a way to solve this puzzle and generalize the solution.

   (b) What is the runtime of your solution?

   (c) Do there exist any unsolvable permutations? Explain.

2. Another puzzle that is fun to figure out is the following:
   - (Stephen) You are given a 3×3 grid with entries that are the numbers $1, 2, ..., 9$. The solved grid looks like this:

1	2	3
4	5	6
7	8	9

   - A scramble will be given to you in this grid as a reordering of these numbers.
   - You are only allowed to reorder the grid by cyclically permuting rows and columns.

   (a) Define notation that allows you to express movements in the puzzle in short hand form. Figure out a way to solve the puzzle, and use your defined notation to express your solution as a set of algorithms that can be read.

   (b) Also, does this puzzle have any unsolvable permutations? (In other words, are there any starting positions in this game from which we cannot reach the solved state via our moves?) Explain.

   (c) Come up with a way of describing this puzzle so that it is in fact the puzzle from problem 2.

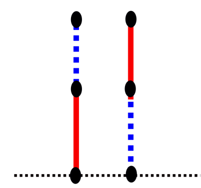
3. (Stephen) Explain how both of the puzzles in problems 1 and 2 form groups (in the same sense that the Rubik's cube formed a group in the talk.)

4. (Eric B.) Take the game Blue-Red Hackenbush which plays as follows: Given a picture comprised of two different colored line segments (Blue and Red), each player alternates removing a segment of a specific color, one color per player. Any line segments not connected to the ground by another segment are removed as well. The first person to be unable to make a move, i.e. have an edge to remove, loses.

   (a) Check each condition of a combinatorial game and determine if Hackenbush is combinatorial or not. Make sure to explain how it satisfies each condition or which conditions it breaks.

   (b) Is this game impartial or partizan? Explain your answer.

   (c) Take the given starting position of a game of Hackenbush:

   Draw a game tree for each possible position of the game. Using this game tree, determine the winning strategy for this starting position of Hackenbush.

5. Consider the following game:

   - Game board: a 5×5 board.
   - Players: 2 players, Horizontal and Vertical. Horizontal goes first.
   - Players alternate taking turns placing dominoes on this board. Specifically, horizontal must place one horizontal 2×1 domino on its turn, and vertical must place one vertical 1×2 domino on its turn.
   - You lose if you cannot place a domino.

   (a) Is this game impartial?

   (b) With perfect play, one of these players should always be able to win on our 5×5 board. Which player can guarantee a win?