Math/CCS 103 — Professor: Padraic Bartlett

Homework 13: Presentations (Taom, Mao and Yukang)

Due Friday, week 7 — *UCSB 2014*

Do **two** of the **five** problems below!

1. (Taom) Indirect self reference is when an object "subtly" refers to itself.

   Jokes are great for getting a (informal) feel of this. For example, an indirectly self-referencing joke would be

   A duck, a frog, and a mathematician walk into a bar. The bartender says "What is this, some kind of joke?"

   Oppose this to a directly self-referencing joke:

   A duck, a frog, and a mathematician walk into a bar. The bartender says "What can I get you?"

   The duck says "Grapes!"

   The frog says "Kiwi!"

   The mathematician hesitates, then glances around, confused. After a moment he says, "I don't think I'm supposed to be in this joke."

   In the direct joke we immediately see the self reference – it is explicit. The indirect joke requires more thought.

   Write two jokes that indirectly self reference themselves and two jokes that directly self reference themselves. Briefly explain why each joke has indirect/direct self reference.

2. (Taom) Quining is an indirect form a self reference and is central to Gödel's proof. To **quine** a phrase is to precede the phrase by itself. For example, take the phrase

   is a sentence fragment

   Quining this gives

   "is a sentence fragment" is a sentence fragment

   Another example is

   "when quined, makes quite a statement" when quined, makes quite a statement

Much like how "this statement is false" creates a paradox when we try to decide if it's true or false, we can use quined statements to create similar paradoxes.

Do this – find a quined statement that creates a paradox.

3. (Taom) The Paddy-Bat problem is the following:

   We have three symbols: P (Paddy), J (the Joker), and B (Batman). These symbols can be combined to form strings under the following axioms:

   (a) The string PJ exists.
   (b) We can add B to the end of any string ending in J. (ex. PJ $\implies$ PJB.)
   (c) We can double the string after the P. That is, we can change Px, to Pxx. (ex. PJB $\implies$ PJBJB.)
   (d) We can replace any JJJ with a B. (ex. PBJJJB $\implies$ PBBB.)
   (e) We can remove any BB. (ex. PBBB $\implies$ PB.)

   Together these axioms form theory 🦇. Any string of the form Px is called a **theorem**. Any string we can create from our axioms is said to be **provable in theory 🦇**. (For example, PJB is provable – we apply axiom 1 and then axiom 2.)

   The question: is the theorem PB provable in theory 🦇?

4. (Mao) Create a 6×6 magic square.

5. (Yukang) In my talk, we discussed the probability that out of n people, there are two that share the same birthday. What is the probability that out of n people, there are **two** pairs of people that share the same birthday? How about **three** pairs of people?