Math/CCS 103 Professor: Padraic Bartlett

# Homework 16: Period Three Implies Chaos; Unit Distance Graph

*Due Friday, week 9* *UCSB 2014*

Do **two** of the **four** problems below!

1. We defined the concept of a **relation** $R : S^2 \to \{T, F\}$ on a set $S$ in class on Monday, and discussed three properties that relations can have:

   - **Reflexivity**: for any $x \in S$, $xRx$.
   - **Symmetry**: for any $x, y \in S$, if $xRy$, then $yRx$.
   - **Transitivity**: for any $x, y, z \in S$, if $xRy$ and $yRz$, then $xRz$.

   Any relation can satisfy or not satisfy each of the above properties, for a total of eight possible combinations $(2 \cdot 2 \cdot 2)$.

   Come up with eight relations, one for each of the eight possible truth triples for reflexivity, symmetry, and transitivity. All of these relations should be on sets that are not numbers and that are definable outside of mathematics (i.e. "x is heavier than y," on the set of planets in our solar system; "x has dunked on y," on the set of players in the NBA; etc.)

2. Prove, as claimed in class today, that if $\frac{a}{b}, \frac{c}{d}$ are two rational numbers written so that $GCD(a, b) = 1 = GCD(c, d)$, then if $a^2d^2 + b^2c^2 = b^2d^2$, we must have that both $b$ and $d$ are odd.

3. Take the collection of all rational points equivalent to $(0, 0)$, under the equivalence relation defined in class. Prove the claim that we made in class: if we color points in this class red when they are of the form $\left(\frac{odd}{odd}, \frac{odd}{odd}\right), \left(\frac{even}{odd}, \frac{even}{odd}\right)$ and color them blue otherwise, then any two points that are distance 1 apart in this class are colored different colors.

4. Find $\chi(\mathbb{Q}^3)$.