Math/CCS 103	Professor: Padraic Bartlett

Homework 7: Triangulations and NP

Due Friday, week 4 *UCSB 2014*

Pick **two** of the problems below, and solve them!

1. In class, we showed that given a formula f from an instance of 3SAT, we can create a graph G such that G has a triangulation if and only if f is satisfiable. Suppose you have a formula f' that's from an instance of 4SAT: i.e. a formula of the form

$$(l_{1,1} \vee l_{1,2} \vee l_{1,3} \vee l_{1,4}) \wedge (l_{2,1} \vee l_{2,2} \vee l_{2,3} \vee l_{2,4}) \wedge \ldots \wedge (l_{n,1} \vee l_{n,2} \vee l_{n,3} \vee l_{n,4}).$$

   Can you create a graph G' that has a triangulation whenever f is satisfiable?

2. Take two $H_{3,n}$'s. Create a way to glue these graphs together such that the following happens:
   - We can completely triangulate either one of these $H_{3,n}$'s.
   - However, doing so makes it impossible to triangulate the other $H_{3,n}$.

3. A 4**-cycle decomposition** is basically a triangle decomposition, except with squares (i.e. 4-cycles): i.e. it is a way to break the edges of a graph into disjoint subsets, each one of which forms a 4-cycle.
   (a) Explain why if a graph has a 4-cycle decomposition, the degree of every vertex must be even and the number of edges must be a multiple of 4.
   (b) Find a graph that has every vertex of even degree and its number of edges a multiple of 4, but does not have a 4-cycle decomposition.
   (c) Find a complete graph K_n that has a 4-cycle decomposition.

4. (Trickier.) Generalize problem 3: for any m, find a n such that K_n has a m-cycle decomposition.

5. (Open problem!) Our $H_{3,n}$ graphs had the following properties
   - Each $H_{3,n}$ graph had exactly two possible triangulations.
   - The degree of every vertex in a $H_{3,n}$ was 6.

   Find a family of graphs G_n such that
   - Each G_n graph had exactly two possible triangulations.
   - There is some constant C, like say 1/2 or 1/6, such that each vertex has degree $C \cdot |V|$.

   In other words, our $H_{3,n}$ graphs were "sparse" graphs, that didn't have many edges, and also had only two triangulations. We want a collection of graphs G such that they are "dense" (i.e. the degree grows linearly in the number of vertices, i.e. every vertex is connected to like 1/8th of the other vertices or something like that) and yet still only have two triangulations.