Math/CCS 103	Professor: Padraic Bartlett

Homework 9: Presentations (Landon and Alice)

Due Friday, week 5 — *UCSB 2014*

Do **two** of the **four** problems below!

1. (Alice) We constructed the Cantor set by deleting a number of intervals from $[0, 1]$: i.e. we started by removing $(1/3, 2/3)$, and then removed $(1/9, 2/9) \cup (7/9, 8/9)$, and repeated this process as described in the talk. This leaves behind the "endpoints" of these intervals in the Cantor set: i.e. $1/3, 2/3, 1/9, 2/9, 7/9, 8/9$ are all in the Cantor set.

   Find a number in the Cantor set that is not one of these "endpoint" points. Explain why your point is not an endpoint of one of the intervals we cut out of the Cantor set. Explan why it is in the Cantor set.

2. Consider the following set:

   - Start with the interval $[0, 1]$.
   - Remove the middle $1/2$ of this set: i.e. remove the set $(1/4, 3/4)$ from our set, leaving the two intervals $[0, 1/4] \cup [3/4, 1]$.
   - Remove the middle $1/2$ of those two sets, leaving $[0, 1/16] \cup [3/16, 1/4] \cup [3/4, 13/16] \cup [15/16, 1]$.
   - Repeat this process.

   Let $\mathcal{D}$ denote the limit of the above process.

   (a) What is the "length" of $\mathcal{D}$, using the notion of length that Alice gave in her talk?

   (b) What do the elements of $\mathcal{D}$ look like in binary? What is a very simple criteria you can give about binary numbers that tells you whether a given binary string is in $\mathcal{D}$?

3. (Landon) Take a circle and inscribe a regular square inside of it. What is the probability that a random chord is longer than a side of our square? Calculate this probability in three different ways (as done in the talk.)

4. Consider the following fourth method for picking a random chord:

   - Pick a random point in the interior of the circle.
   - Pick a second random point in the interior of the circle.
   - Draw the chord through those two points.

   How likely is it that this chord is longer than the base of an inscribed equilateral triangle?