Cellular Automata

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What is a Cellular Automaton?

Definition: A **cellular automaton** is a modeling system that consists of an n-dimensional grid of cells. Each of these cells have an **initial state** ie. dead or alive, on or off, etc.

These cells are then updated with some given set of **rules** to model different situations.

More on Cellular Automaton

2-D Grid: For simplicity's sake, we are going to consider cellular automata over $\mathbb{Z}^2$, i.e. a 2-dimensional grid. In this grid, we are going to define a concept of **neighborhood**: i.e. for any given cell (x, y), the collection of all of the other cells whose life/death status "matters" to cell (x, y).

Moore Neightborhood: The study of a cell's eight neighbors, which consists of all the surrounding cells.

Question: how many different kinds of Moore neighborhood can a cell have?

von Neumann Neighborhood: The study of a cell's adjacent neighbors, ie. cells that differ from (x, y) in one coordinate, and whose distance from (x, y) is 1.

Question: which cells do these consist of?

Conway's Game of Life

- Conway's Game of Life is a classic example of a cellular automaton. It is a "zero" player game, ie. the game's evolution is depended only upon the initial state of the cells.
- Typically, this game considers the cell's Moore neighborhood under a certain sets of rules...

Rules

- Any live cell with fewer than two live neighbors dies, as if caused by under-population.
- Any live cell with two or three live neighbors lives on to the next generation.
- Any live cell with more than three live neighbors dies, as if by overcrowding.
- Any dead cell with exactly three live neighbors becomes a live cell, as if by reproduction.

Java Simulation:

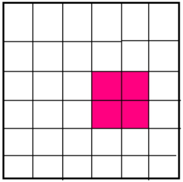
http://www.newgrounds.com/portal/view/493239

Still Life, Oscillators and Spaceships

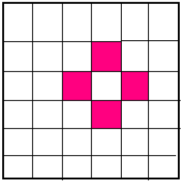
Still Life: A formation of live cells that stays constant from some time t until infinity, unless the state of the neighbors are changed.

Examples: From the simulation, what are some pictures of still life formations?

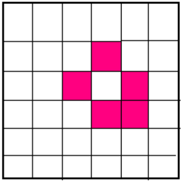
block



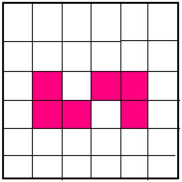
tub



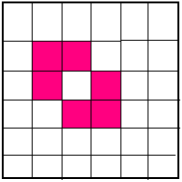
boat



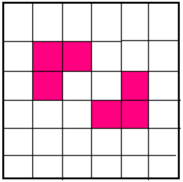
snake



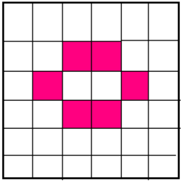
ship



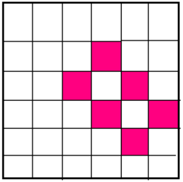
aircraft carrier



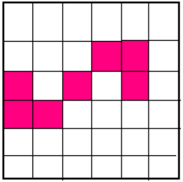
beehive



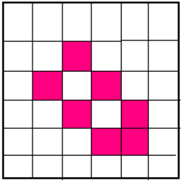
barge



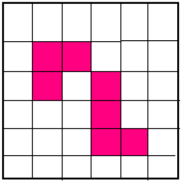
Python



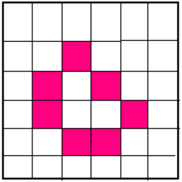
long boat



fish hook



loaf



Still Life, Oscillators and Spaceships Cont.

Oscillators: A pattern that returns to its original state after some **period** (number of generations), in the same orientation and position, after a finite number of generations. http://en.wikipedia.org/wiki/Oscillator_(cellular_automaton)

Still Life, Oscillators and Spaceships Cont.

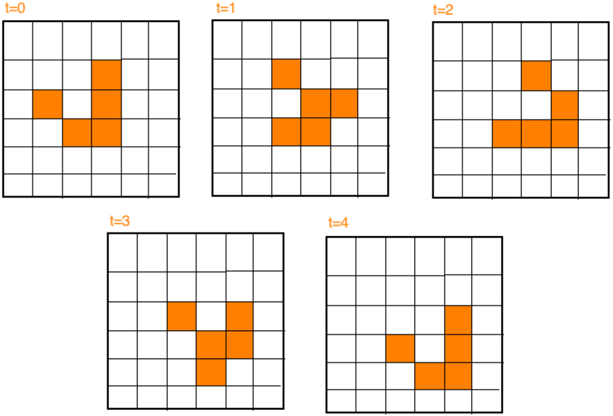
Spaceships: A finite pattern that returns to its initial state after a certain period, but in a different location.

In some literature, spaceships are referred to as gliders or fish.

Examples of Spaceships

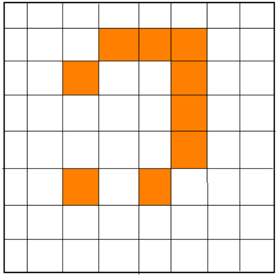
Glider: The smallest and most common type of spaceship. It travels southeast and repeats its original form every 4 generation (has period 4).

Glider

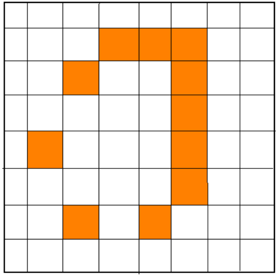


Spaceships with Various Weights

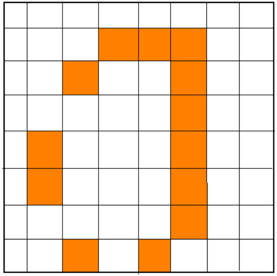
Light Weight Space Ship



Medium Weight Space Ship



Heavy Weight Space Ship



Notion of Speed

Definition: The **speed** of a spaceship is the number of cells that the pattern moves during its period divided by the period length.

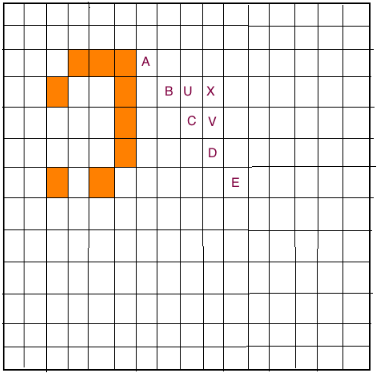
Notation: We want to study the notion of speed compared to c, the speed of light in physics. Here, c is reached when the pattern moves one cell in one generation. In a single generation, a cell can only influence its nearest neighbors, and thus the speed of light is the maximum rate at which information can propagate. It is therefore an upper bound to the speed at which any pattern can move.

More on Speed

Gliders' Speed: A glider travels diagonally at the speed $\frac{c}{4}$ because it travels one cell per 4 generations.

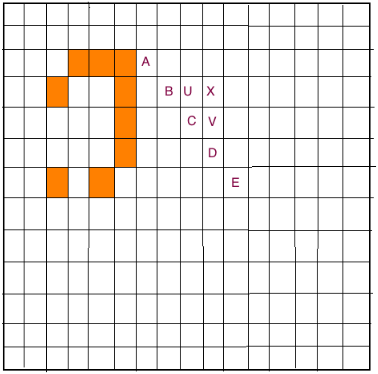
Spaceships' Speed: A light weight spaceship travels orthogonally in any direction $\frac{c}{2}$ because it moves over a cell every other generation.

Theorem: No spaceship can travel diagonally faster than $\frac{c}{4}$.



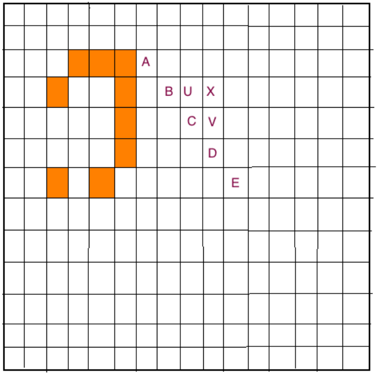
Proof:

- Suppose the spaceship is in this position at generation 0.
- Suppose that cell X will be alive in generation 2. Then cells C, U, and V must have been alive in generation 1. This means that U and V must have had 3 alive neighbors in generation 0, so each of B, C, D, J, and K must be alive in generation 0.



Cont.

- This means that C must have at least four live neighbors in generation 0 though, so there is no way for it to survive to generation 1, which gives a contradiction.



Cont.

- It follows that X can not be alive in generation 2.
- In other words, if the spaceship is behind the diagonal line A, B, C, D, E in generation 0, then it must be behind the diagonal line defined by U and V in generation 2. It follows that can not travel faster than $\frac{c}{4}$ diagonally.

Homework Problem

Theorem: No spaceship can travel orthogonally faster than $\frac{c}{2}$.

Prove it!

References:

http://web.mit.edu/sp.268/www/2010/lifeSlides.pdf

http://en.wikipedia.org/wiki/Conway's_Game_of_Life

http://en.wikipedia.org/wiki/Cellular_automaton

http://mathworld.wolfram.com/CellularAutomaton.html

http://www.conwaylife.com/wiki/Spaceship