Random Walk Problem

Tianruo Chen(Roy)

May 9, 2014

What Random Walk Problem is?

Drunkard's walk

- A random walk is the process by which randomly-moving objects wander away from where they started.
- Lets model a drunkards very simplified map of the universe:
- Here are only four possible position: H: The drunkards home; B: a black-hole can absorb everything. And two intermediate point x and y.

Drunkard's walk

When drunkard is not at point H or B. He needs to flip a coin to decide where he will go next. If drunkard goes home at last, he will be safe and sleep. If drunkard goes to the black hole, he will disappear forever. What is the probability of the drunkard to go home safely if he starts at point x?

Drunkard's walk

- Lets consider the drunkard start from any possible vertex. And use p(v) to denote the possibility of drunkard to go home successfully.
- $P(H)=1$
- $P(B)=0$
- $P(x)=1/2P(H)+1/2P(y)$
- $P(y)=1/2P(x)+1/2P(B)$

Drunkard's walk

- Lets consider the drunkard start from any possible vertex. And use p(v) to denote the possibility of drunkard to go home successfully.
- $P(H)=1$
- $P(B)=0$
- $P(x)=1/2P(H)+1/2P(y)$
- $P(y)=1/2P(x)+1/2P(B)$
- We can get $p(x)=1/3$ $P(y)=2/3$ by solving those function out.

Drunkard's Walk

- Let's use matrices to denote the possibility.
- In each cell (i,j), we put the probability that we will go to position i from position j.

Drunkard's walk

- However, a drunkard cant always flip the coin to decide where he goes. He is more likely to go to a more clean and light road. We can attach weights Wx,y to every edge in the graph, that denotes the likelihood a drunk pick that road over other road available to it.
- $P(x) = \sum_{y \in neighboursofx} P(y) \cdot \frac{W_{xy}}{W_x}$
- Here
- $W_x = \sum_{y \in neighboursofx} W_{xy}$

Matrix Method

- We can use matrix to help us predict where the drunkard will go.
- Assume the drunkard have 8 intermediate point between home and black hole. We use $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$to denote it. We H,$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$,B. The possibility of the coin that the drunkard use is about 0.51 for head to go left and o.49 for tail to go right.

Matrix Method

- Assume we start from A_5.We can use A=(0,0,0,0,0,1,0,0,0,0,0) to multiple the matrix we get before. We will get AP=(0,0,0,0.59,0,0.41,0,0,0,0), it shows the possibility of drunkard go to point A_4 and A_6. And A^2P=(0,0,0,.2601,.4998,.2401,0,0,0) shows the possibility of drunkard to go A_3, A_5, A_7.
- We want to calculate An for large power of n. When n=100, we will find A^{100}P≈(.55,0,0,0,0,0,0,0,0,.45) in the end. The possibility of the drunkard to go back home is about 0.55.

Random Walk on Z

- we are taking a random walk on Z, where we start at the origin, each number n is connected to n-1 and n+1, and we are as likely to walk in the +1 direction as the -1 direction.
- Suppose we put the black dot at 0 and then let it take N steps (where N is any number). Now we want to know how far the black dot travels after it has taken N steps
- $d = a_1 + a_2 + ... + an$
- we use $\langle d \rangle$ to denote the average distance
- $\langle d \rangle = \langle a_1 + a_2 + ... + an \rangle = \langle a_1 \rangle + \langle a_2 \rangle + ... + \langle a_n \rangle$

Random Walk on Z

- But $\langle a_1 \rangle$=0, because if we repeated the experiment many many times, and a_1 has an equal probability of being -1 or +1, we expect the average of a1 to be 0. So then
- $\langle d \rangle = \langle a_1 \rangle + \langle a_2 \rangle + ... + \langle a_n \rangle = 0 + 0 + ... + 0 = 0$

Random Walk on Z

- It's useless. Let's calculate $\langle d^2 \rangle$ because it will never be 0.
- $\langle d^2 \rangle = \langle a_1 + a_2 + ... + an \rangle^2 =$
  $\langle a_1 + a_2 + ... + an \rangle \times \langle a_1 + a_2 + ... + an \rangle =$
  $(\langle a_1^2 \rangle + \langle a_2^2 \rangle + ... + \langle a_n^2 \rangle) +$
  $(\langle a_1 a_2 \rangle + \langle a_1 a_3 \rangle + ... + \langle a_{n-1} a_n \rangle) =$
  $(1 + 1 + ... + 1) + (0 + 0 + 0 + 0 + ... + 0) =$
  N
- $\sqrt{\langle d^2 \rangle} = \sqrt{N}$ it is something like the average positive distance away from 0 after N steps.

THAT IS All

- Thank you for lsitening