Math/CS 103	Professor: Padraic Bartlett

Handout 10: Magic Squares

Week 5	*UCSB 2014*

Pick **two** of the **four** problems below, and solve them!

1. Consider the following construction:

   Construction. Take any value of n, and any two numbers $a, b \in \{0, \ldots n-1\}$. Consider the following square populated with the elements $\{0, 1 \ldots n-1\}$:

   $$L = \begin{array}{|c|c|c|c|c|c|} \hline 0 & a & 2a & 3a & \ldots & (n-1)a \\ \hline b & b+a & b+2a & b+3a & \ldots & b+(n-1)a \\ \hline 2b & 2b+a & 2(b+a) & 2b+3a & \ldots & 2b+(n-1)a \\ \hline 3b & 3b+a & 3b+2a & 3(b+a) & \ldots & 3b+(n-1)a \\ \hline \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline (n-1)b & (n-1)b+a & (n-1)b+2a & (n-1)b+3a & \ldots & (n-1)(b+a) \\ \hline \end{array} \mod n.$$

   In other words, L's (i, j)-th cell contains the symbol given by taking the quantity $ai + bj \mod n$.

   Determine rules on a, b that determine when this square is a diagonal Latin square. (Try some small cases!)

2. Suppose that L is a diagonal Latin square made by the above process. Show that L^T, the **transpose**[1] of L, is another diagonal Latin square. Furthermore, show that L^T is orthogonal to L.

3. Take any pair of orthogonal diagonal $n \times n$ Latin squares L_1, L_2 on the symbols $\{0, \ldots n-1\}$. Create the square M as follows: if the cell (i, j) contains the symbol x in L_1, y in L_2, write down the number $nx + y$ in the cell (i, j) of M. Prove that this square M is a magic square.

4. Construct a magic square of order 5 by using the methods above. Furthermore, prove that the methods above do not create **every** possible magic square: i.e. find some magic square M that cannot be created by the methods above.

[1] The **transpose** of a matrix is t