In this handout, we are studying the following question: what classes of partial Latin squares always have completions?

Specifically, consider the following definition:

Definition. An order n **Latin rectangle** is a $n \times n$ partial Latin square P, such that the first k rows of P are completely filled in and the remaining $n - k$ are completely blank.

For example, the following is an order 4 Latin rectangle with three completed rows:

1	2	3	4
2	1	4	3
3	4	1	2

Before we get to the real problems this week, a warm-up (not turned in) to check the definitions:

Question 1. *Let P be an order n Latin rectangle consisting of $n - 1$ filled rows. Show that P can be completed to a Latin square.*

Once you do this, attempt the following three questions. These all should be written up in LATeX and turned in on **Friday**.

1. Let P be an order n Latin rectangle consisting of 1 filled row. Show that P can be completed to a Latin square.

2. Let P be an order n Latin rectangle consisting of $n - 2$ filled rows. Show that P can be completed to a Latin square.

3. Let P be an order n Latin rectangle consisting of 2 filled rows. Show that P can be completed to a Latin square.