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Handout 4: Bipartite Graphs

Week 2 *UCSB 2014*

In this handout, we are studying more questions about **graphs**! In particular, we're returning to **bipartite graphs**: graphs that can be split into two sets V_1 and V_2, such that there are no edges in G connecting two vertices in V_1, or connecting two vertices in V_2 (i.e. all edges involve exactly one vertex in V_1 and one in V_2.)

In particular, consider the following definition:

Definition. A **matching** of a graph G is a collection of edges M, such that every vertex of G is contained in at most one edge in M. Finally, a **perfect matching** of a graph $G = (V, E)$ is a collection of edges P such that every vertex $v \in V$ is contained in precisely one edge in P.

There are only two questions I want us to do today. As always, LATeX up your work, and be able to turn it in by **Wednesday**, January 22.

1. Take a bipartite graph $G = (V, E)$ with bipartition $V = V_1 \cup V_2$. Suppose that there is some number k such that the degree of every vertex in G is k. Then G has a perfect matching: i.e. there's a way to pair off all of the elements of A with the elements of B.

2. Using problem 1, prove that every Latin rectangle can be completed to a Latin square.