| Math/CS 103 | Professor: Padraic Bartlett |
|---|---|

# Handout 5: Finite Fields

*Week 2* *UCSB 2014*

In this handout, we are studying finite fields! We discussed finite fields in the YouTube lecture; this handout is meant to test out some of those ideas. As always, LATeX up your work, and be able to turn it in by **Wednesday**, January 22nd, along with the other sets due that day.

1. Suppose that $F$ is a finite field. Prove, using only the axioms that define a field that we discussed in class, that there is only one multiplicative identity 1 in $F$. In other words: suppose that $F$ is a field, and that you have two elements $1, e$ such that

$$1 \cdot x = e \cdot x = x,$$

   for any $x \in F$. Prove that $1 = e$.

2. Prove that $\langle \mathbb{Z}/n\mathbb{Z}, +\cdot \rangle$ is a finite field whenever $n$ is prime, and not a field whenever $n$ is not prime.

3. Find a finite field containing exactly four elements.

   Hint: $\mathbb{Z}/4\mathbb{Z}$ is not an example, as you've just proven in problem 1! So you have to try something else. Specifically: see if there is a way to fill in the addition and multiplication tables

| $+$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | | | | |
| $b$ | | | | |
| $c$ | | | | |
| $d$ | | | | |

,

| $\cdot$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | | | | |
| $b$ | | | | |
| $c$ | | | | |
| $d$ | | | | |

   in such a way that you get something that's a field!